The use of a Subgrid-Scale Quasi-Stationary Approximation in Finite-Difference Time-Domain Simulations to Calculate the Absorption by Helmholtz Resonators

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Abstract: The viscous absorption in Helmholtz resonators was calculated using Finite-Difference Time-Domain (FDTD) simulations. To minimize the computational cost, a coarse FDTD grid was used. The FDTD equations in the neighborhood of the resonator neck were corrected for subgrid-scale effects like the viscous absorption in the boundary layers and the strong inhomogeneity of the pressure field near the resonator neck. The simulated spectral dependency of the absorption coefficient was in good agreement with experimental data.

INTRODUCTION

To accurately calculate absorption effects in Helmholtz resonators using a wave-equation based simulation technique like Finite-Difference Time-Domain (FDTD), one is confronted with a scale problem. Both the resonance frequency and the acoustical dissipation are determined by the resonators' geometry which is significantly smaller than the wavelengths of interest. Therefore, the FDTD cell size has to be smaller than normally required for accurate wave propagation simulations. This fact results normally in high computational costs. However, these costs can be minimized using a coarse FDTD grid with some corrected FDTD equations in the neighborhood of the resonator mouth (1). In this paper, we will extend this technique to viscous absorption in the resonator neck which is an important dissipation mechanism in resonators (2).

QUASI-STATIONARY SOLUTIONS

Accurate simulations of wave propagation in a sub-wavelength geometry like a resonator mouth require a fine spatial FDTD grid. This FDTD grid can be avoided when the quasi-stationary pressure distribution nearby the small geometry is first calculated using the Laplace equation for some well-chosen boundary conditions. For an aperture, these boundary conditions are the pressures before and after the aperture (1). Correction terms for the FDTD equations in the coarse grid were extracted out of the quasi-stationary pressure distribution. The accuracy of this technique is mainly determined by the accuracy of the Laplace solutions. The FDTD correction coefficients were calculated for the resonator mouths of interest.

VISCOSITY IN FDTD

The viscous boundary layer is for most applications significantly smaller than the FDTD cell size. Therefore, its influence is averaged over the FDTD cell which results in a dissipative term in the linearized Euler equation for the velocity components parallel to the layer boundary. In the staggered grid FDTD formalism, the FDTD velocity components are averaged over the grid cell surfaces with area $S$. The Euler equation for these averaged velocity components (e.g. $<v_x>$) can be written as

$$\rho \frac{\partial <v_x>}{\partial t} = -<\frac{\partial p}{\partial x}> + U \sqrt{\frac{\mu}{\rho \pi}} \left[ \left\{ \frac{\partial p}{\partial x}(t) \right\}_{n} \right]_{n} \frac{1}{\sqrt{r_i - r}} \text{d}t,$$

where $<\frac{\partial p}{\partial x}>$ is the $x$ component of the mean pressure gradient in the FDTD cell containing the boundary layer, $[\frac{\partial p}{\partial x}]_{n}$ is the $x$ component of the pressure gradient at the border of the viscous boundary layer, $U$ is the circumference of the intersection between the boundary layer and the velocity surface and $\mu$ and $\rho$ are the air viscosity coefficient and the air density, respectively.

After spatial and time discretization of the derivatives, the corrected FDTD equation for the velocity component is obtained. Notice that the extra term is a convolution of the spatial pressure derivative and the time function $t^{1/2}$. In an FDTD algorithm the calculation of the convolution requires the whole time history of these derivatives. For this application, where the number of viscosity cells is very limited, we used this approach. A
less memory-demanding alternative for this problem was worked out in (3), where the convolution was replaced by a recurrency relation. However, the accuracy of this approach is only good in a limited frequency interval.

When no subgrid-scale geometries are present, \( \frac{\partial p}{\partial x} \rvert_{BL} \) in Eq. (1) can be approximated by \( \langle \frac{\partial p}{\partial x} \rangle \). This approximation is not correct for the pressure gradients inside and nearby the resonator circular aperture. However, it can be proven that the ratio \( \frac{\partial p}{\partial x} \rvert_{BL} / \langle \frac{\partial p}{\partial x} \rangle \) is constant and can be calculated using the quasi-stationary solutions. This ratio was calculated for the relevant velocity components near the aperture surfaces.

RESULTS IN HELMHOLTZ RESONATORS

To test the FDTD accuracy for viscous dissipation simulation, the spectral dependency of the absorption coefficient for several Helmholtz resonators placed at the end of a duct was calculated. The results were compared to the semi-empirical analytical formulas of (2). The resonators consist of a circular aperture (radius = 0.77 mm) and a volume of 0.0000164 m\(^3\). The results shown in this manuscript concern a resonator neck length of 1.6 mm. To avoid interaction between the tube and the resonator, the tube radius was chosen 20 times larger than the aperture radius (5). The calculations were performed in a cartesian cubical FDTD grid with cell size of 2 mm.

In Fig. 1, the semi-empirical curve of (2) is compared to the calculated FDTD curve. There is a good agreement between both results.

FIGURE 1. Comparison between the semi-empirical (2) and the simulated absorption coefficient.

CONCLUSIONS

The viscous absorption in Helmholtz resonators can be efficiently included in FDTD simulations using a combination of subgrid-scale quasi-stationary approximations and the viscous boundary-layer theory.

REFERENCES