A modal approach for the acoustic and vibration response of an elastic cavity with absorption treatments and mechanical/acoustical excitation

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Abstract: The calculation of the acoustic and vibration response of an elastic cavity is considered. The presented technique uses a modal decomposition for the structure in vacuo and the cavity to re-write the coupled equations and account for the impedance condition at the treated elastic walls, internal acoustic sources and mechanical excitations in terms of a compact forced system. When no impedance conditions are considered, the coupled equations lead to a coupled symmetric and compact eigen problem in terms of structure and cavity modal coordinates.

INTRODUCTION

The evaluation of the forced vibration and acoustic response of an elastic cavity with absorption treatments is considered. The cavity is excited through both mechanical forces and internal acoustic sources. The general equations are derived using the variational principle and solved using the finite element method. For the treated cavity, the non coupled cavity and structure modes may be used, classically, to solve the problem. However, problems may arise in choosing the number of kept modes when modal coupling becomes important. The use of coupled modes may alleviate this difficulty. An approach is presented to calculate the coupled modes. It is used to evaluate efficiently the forced response for the untreated case. The limitations and advantages of using this coupled base to solve the problem with treatment will be discussed in the oral presentation.

THEORY

Let us consider the problem of the elastic cavity with partial absorption treatments depicted in figure (1). The cavity is treated on surface Σ₂ and not treated on surface Σ₁. The treatment is constituted from a porous-screen system. The structural domain is denoted by Ω_s and the cavity domain by Ω_c. Both mechanical forces on the surface Σ and sources in the domain Ω_c are considered. The acoustic pressure inside the cavity is then written as the sum p = p_s + p_c, where p_s is the pressure generated by the sources in the free field and p_c is the pressure due to the presence of the cavity. The pressure p_c must satisfy the Helmholtz equation and the total pressure p must satisfy the Euler equation on the surfaces Σ₁ and Σ₂. Also, the pressure p is related to the normal displacements on the surface Σ₂ by the relation p = ω²mₐuₐ + jωZ(uₐ - uₙ) where mₐ is the mass per unit area of the limp screen, Z is the impedance of the acoustic layer, uₐ is the normal displacement on the cavity side and uₙ is the normal displacement of the structure. Finally, considering the equation of motion of the structure and the pressure loading on surfaces Σ₁ and Σ₂, the coupled discretized system is given by:

\[
\begin{bmatrix}
K - \omega^2 M + j\omega Z & -j\omega Z & C_1 \\
-j\omega Z^T & -\omega^2 M_c + j\omega Z & C_2 \\
C_1^T & C_2^T & 1/\omega^2 (H - Q)
\end{bmatrix}
\begin{bmatrix}
u \\
u_c \\
p
\end{bmatrix} = \begin{bmatrix}
F - C_1 p_s \\
-C_2 p_s \\
1/\omega^2 S
\end{bmatrix}
\]

where K and M are the structure stiffness and mass matrices, H and Q are the kinetic and compression fluid matrices, C₁ and C₂ are the coupling matrices respectively on surface Σ₁ and Σ₂, M_c is the mass matrix of the septum (limp screen), Z is the impedance matrix of the acoustic layer, u is the structure displacement vector, u_c is the cavity displacement vector on the absorbant, p_s is the cavity pressure vector
and $p_s$ is the source pressure vector. The second equation of (1) can be used to express $u'$ as a function of $u$ and $p$. For instance, in the special case where there is no limp screen, the symmetry of $Z$ can be used to rewrite equation (1) in the following form:

$$\begin{bmatrix} K - \omega^2 M & -C^T \\ -\omega^2 C & H - \omega^2 Q \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} F - C p_s \\ \omega^2 S + A p_s \end{bmatrix}$$

(2)

with $C = C_1 + C_2$ and $A = \frac{1}{j \omega} C_1^T Z^{-1} C_2$.

**STRUCTURE-CAVITY COUPLING**

In this section, the absorption treatments are not considered. It is shown that coupled structure-cavity modes can be found and used to evaluate the forced response of the system. Since no impedance condition is used, equation (2) is reduced to:

$$\begin{bmatrix} K - \omega^2 M & -C^T \\ -\omega^2 C & H - \omega^2 Q \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} F - C p_s \\ \omega^2 S + A p_s \end{bmatrix}$$

(3)

The system is not symmetric and the classical algorithms for the eigen problem can not be used. Reference (1) proposed a method based on the inversion of matrix $H$ to symmetricize the problem. Unfortunately, matrix $H$ is singular and can not be easily inverted. Reference (2) proposed another method, based on a modal approach, to circumvent the non symmetric shape of equation (3). The unknown $u$ and $p$ are written in terms of modal coordinates, $u = \Phi_s q_s$ and $p = \Phi_c q_c$. The modal matrices $\Phi_s$ and $\Phi_c$ are chosen such that:

$$\Phi_s^T M \Phi_s = I_{n \times n} \quad \Phi_c^T Q \Phi_c = \begin{bmatrix} 1 \\ \Lambda_c^{-1} \end{bmatrix} \quad \Phi_c^T H \Phi_c = \begin{bmatrix} 0 & 0 \\ 0 & I_{m \times m} \end{bmatrix}$$

(4)

where $\Lambda_c$ is a diagonal matrix containing the square natural frequencies of the structure, and $\Lambda_c$ is a diagonal matrix containing the square natural non zero frequencies of the fluid. After condensation of the cavity rigid body mode, the linearized equation (3) leads to the final set of equations:

$$\begin{bmatrix} \Lambda_s - \omega^2 I + C_r^T C_s + C_e^T \Lambda_c C_e & -C_e^T \Lambda_c \\ -C_e \Lambda_c & \Lambda_c - \omega^2 I \end{bmatrix} \begin{bmatrix} q_s \\ q_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(5)

where $\tilde{q}_c = \frac{1}{\omega^2} q_{c,c}, C_r = \Phi_c^T C \Phi_s, C_e = \Phi_c^T C \Phi_s$, and where the subscript “e” stands for elastic and “r” for rigid. Once system (5) solved, the coupled modes of the system are given by:

$$\Phi_{sc} = \begin{bmatrix} \Phi_s \\ -\phi_{cr} C_r - \Phi_c \Lambda_c C_e \\ \Phi_c \Lambda_c \end{bmatrix} V$$

(6)

where $V$ are the solutions of equation (5). The solution of the forced system (3) is then given by:

$$\begin{bmatrix} u \\ p \end{bmatrix} = \Phi_{sc} q - \begin{bmatrix} 0 \\ \phi_{cr} \phi_{cr}^T + \Phi_c \Lambda_c \Phi_{sc} \end{bmatrix} \begin{bmatrix} 1 \\ \omega^2 S + A p_s \end{bmatrix}$$

(7)

where $q$ is solution of $(\Lambda_s - \omega^2 I) q = \Phi_{sc}^T \begin{bmatrix} F - C p_s \\ \omega^2 S + A p_s \end{bmatrix}$.

The oral presentation discusses the limitations and advantages of using the coupled modes to solve efficiently equation (1).

**ACKNOWLEDGMENTS**

This work was funded by the IRSST (Institut de Recherche en Santé et Sécurité au Travail), Canada.

**REFERENCES**
