A Modal formulation for an arbitrary submerged structures in a heavy acoustic medium

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Abstract: The paper studies the radiation and dynamic characteristics for an arbitrary submerged interacting with the surrounding acoustic medium. The structural equation coupled with the surrounding fluid loading is formulated in a symmetric complex matrix equation under mono-frequency oscillations. As a result of such symmetric formulation, an eigenvalue problem is emerged as one considers stationary conditions for the ratio of the real part of the complex power exerted by the external force to the imaginary part of the complex power. The eigensolutions in turns decouple the coupled equation such that impedance equation associating with each eigensolution each fully describes the radiation and dynamic characteristics. An effective modal admittance is identified for the efficiency of the structure radiation acoustic power associated with each eigenmode.

INTRODUCTION

One of the interesting and practical problems in structural acoustics is to understand the radiation and dynamic characteristics of submerged structures in a fluid. The characteristics characterize radiation and dynamic responses of the coupled system in which the structural equation interacts the acoustic loading. The identification of the characteristics is founded on a symmetric formulation of the coupled system where the acoustic loading is a symmetric complex matrix derived by a previous established surface acoustic reciprocity [1]. An intimate relationship with the present analysis is the subject of radiation mode. Radiation modes were proposed by Borgiotti [2], Photiadis [3], and Sarkissian [4] etc. in the years of 1990, decoupling the surface normal velocity into modal representations where each mode has its own radiation efficiency to radiate acoustic power. Another type of radiation, derived from a different viewpoint from the above, was established based on a surface complex acoustic power. The modal representation developed by the present analysis, termed as a coupled structural/acoustic radiation mode, will be used to interpret resonant phenomenal for submerged elastic structures.

MODAL FORMULATION for A SUBMERGED ELASTIC STRUCTURE

A submerged elastic structure vibration at a mono-frequency. We can have the equation of motion,

\[
\begin{align*}
[D_{nn}][X_n]+[D_{ni}][X_i]+[h](i\omega)[X_n] &= \{f_n\} \\
[D_{in}][X_n]+[D_{ii}][X_i] &= \{f_i\}
\end{align*}
\]

(1)

where the subscripts "n" denote the degree of freedom outward normal to the wetted surface of the structure, the subscripts "i" denote the rest degrees of freedom of the structure which include displacements tangent to the wetted surface and the degrees of freedom of the internal structural members \(\{X_n\}\) and \(\{X_i\}\) are the displacements, \(\{f_n\}\) and \(\{f_i\}\) are the corresponding amplitudes of alternative applying forces, \([h]\) is a complex symmetric acoustic loading matrix, \([h]=-[h_n]+i[h_n]\), and \([D_{pq}]\) (p,q = i,n) is defined by the mass matrices \([M_{pq}]\) and stiffness matrices \([K_{pq}]\)

\[
[D_{pq}]=[K_{pq}]-\omega^2[M_{pq}], \quad p,q = i,n.
\]

(2)

The velocity variables \(v_n\) and \(v_i\) relate to the displacements, \(v_n = i\omega X_n\) and \(v_i = i\omega X_i\). The ratio \(\eta\) of the real part to imaginary part of the complex power exerted by forces \(f_n\) and \(f_i\) is

\[
\eta = \frac{\nu_n^T h \nu_n}{\nu_i^T h \nu_i} - \frac{K_{nn} v_n - K_{ni} v_i + h_i v_n}{\omega} + i \left( -\frac{K_{nn} v_n - K_{ni} v_i}{\omega} v_i \right).
\]

(3)

The ratio \(\eta\) becomes a large value when the external forces and responded velocity are more in-phase.
while a small value of $\eta$ indicates that the phase tends to $90^\circ$. This ratio $\eta$ depends on the velocities of $v_n$ and $v_i$, which suggests considering stationary values for $\eta$ when the velocities $v_n$ and $v_i$ are varied. This yields a symmetric eigenvalue problem at stationarity, that is,

$$
\begin{bmatrix}
[h_R] & 0 & [v_n] \\
0 & 0 & [v_i]
\end{bmatrix} = \eta \begin{bmatrix}
-\frac{[K_{nn}]}{\omega} + [h,] & \frac{[K_{ni}]}{\omega} \\
\frac{[K_{in}]}{\omega} & \frac{[K_{ii}]}{\omega}
\end{bmatrix} \begin{bmatrix}
[v_n] \\
[v_i]
\end{bmatrix}.
$$

(4)

The forced vibration of the coupled structural/acoustic system described in Eq.(1) can be investigated by the coupled structural/acoustic radiation mode. Using the second equation of Eq.(1) to eliminate the velocity variable $\{v_i\}$ from the first equation leads to

$$
[h_R][v_n] + i[H][v_n] = \hat{f}_n,
$$

(5)

where the matrix $H$ is identified as

$$
[H] = -\frac{[K_{nn}]}{\omega} + \frac{[K_{ni}]}{\omega} [K_{in}]^{-1} + [h,],
$$

(6)

and the external force $\{\hat{f}_n\}$ is defined by

$$
\{\hat{f}_n\} = \{f_n\} - [D_{nn}][D_{ni}]^{-1}\{f_i\}.
$$

(7)

The velocity variable $\{v_i\}$ is related to the variable $\{v_n\}$ by

$$
\{v_i\} = \left(\frac{[D_{ii}]}{i\omega}\right)^{-1} \{f_i\} - \frac{[D_{ni}]}{i\omega} \{v_n\}.
$$

(8)

As a result of the above derivation, the coupled equation can be decoupled into a set of modal impedance equations,

$$
\left(P_{R,ii} + i\frac{P_{R,i}}{\eta_i}\right)\hat{a}_i = (\Phi)[V_i]^T\{\hat{f}_n\},
$$

(9)

where the $P_{R,ii}$ is the radiated modal acoustic power associated with the $i$th mode, and $\eta_i$ is the corresponding stationary value. The values of $P_{R,ii}$ and $\eta_i$ characterize the radiation and dynamic responses of the submerged structure.

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**REFERENCES**