Abstract: A two-dimensional cavity enclosed by rigid walls and a flexible beam has been studied. The focus has been on the coupled modes. The present example considers a clamped beam covering a hard-walled cavity. It is shown that the finite difference technique, using difference expressions with errors of the order mesh size squared, is well suited to solve the governing system equations.

INTRODUCTION

Elastic plates covering cavities are common in engineering structures. The resonant frequencies of such coupled systems are therefore of interest. They are in general different from the ones of their separate sub-systems. In particular, shallow cavities might represent stiffnesses superior to the ones of the elastic sub-structures, with a considerable influence on the natural frequencies. For regular geometries, analytical methods might be used; modal methods are presented by for instance Hong and Kim in [1]. Numerical methods, like the finite difference method, have the advantage that more complex geometries might be handled, and that local boundary conditions might be written. The errors involved when doing finite difference calculations can be estimated from the Taylor series the finite difference expressions are based on. Such errors can be minimized by choosing high order finite difference expressions, or employing schemes like Richardson extrapolation [2].

THEORY

The beam and cavity are meshed and aligned a Cartesian coordinate system. \( m \) and \( n \) designate points \( \Delta x \) and \( \Delta y \) apart in the \( x \) and \( y \) directions. The beam and Helmholtz equations are written for all the mesh points. The finite difference expressions are written with an error of order \( e = O(\Delta x^2) \). For a point on the beam \((m,n) = N\):

\[
EI \left( \eta_{m-2,N} - 4\eta_{m-1,N} + 6\eta_{m,N} - 4\eta_{m+1,N} + \eta_{m+2,N} \right) / \Delta x^4 + \mu \omega^2 \eta_{m,N} + p_{m,N} = 0 \tag{1}
\]

while for a point \( m, n \) in the cavity:

\[
\frac{p_{m-1,n} - 2p_{m,n} + p_{m+1,n}}{\Delta x^2} + \frac{p_{m,n-1} - 2p_{m,n} + p_{m,n+1}}{\Delta y^2} + k^2 p_{m,n} = 0 \tag{2}
\]

\( \eta \) and \( p \) are the beam displacements and cavity pressures. \( \mu \) and \( \rho \) are the surface and mass densities of the structure and acoustic fluid, and \( k \) is the wavenumber \( \omega / c \) where \( c \) is the speed of sound. \( EI \) is the beam's bending stiffness.

Note that the beam equation incorporates the pressure, while the Helmholtz equation is written in its homogeneous form. In addition, the beam velocity is equated the acoustic particle velocity at the beam/cavity interface.

\[
\frac{1}{j \omega \rho} \frac{p_{m,N+1} - p_{m,N-1}}{2 \Delta y} = j \omega \eta_{m,N} \tag{3}
\]
This last relation involves the points shown to the right below. As is seen, in addition to being a necessary boundary condition, it gives us the means to eliminate the pressure value at the “out of domain” point \( m,N+1 \) created when writing the Helmholtz equation for a point \( m,N \). Points outside the rigid cavity walls and the beam supports are eliminated in similar ways.

\[
\begin{align*}
&\eta_{0,m} = 0 \\
&\eta_{1,m} = \eta_{1,N}
\end{align*}
\]

**FIGURE 1.** Boundary condition for the clamped beam, and the mesh points for equation 3.

## RESULTS

The following table shows the calculated modes of a particular situation of weak coupling (10mm thick, 1.0m long clamped steel beam, air cavity of depth 0.5m). The table indicates the dominant motions of the two sub-systems, as well as their analytical wavenumber values. The coupled wave number (column 4) was Richardson extrapolated from a 12x6 and a 24x12 system.

<table>
<thead>
<tr>
<th>mode nr.</th>
<th>structure</th>
<th>cavity</th>
<th>( k )</th>
<th>( k_x )</th>
<th>( k_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \lambda/2 )</td>
<td></td>
<td>0.98465</td>
<td>0.9750</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( \lambda )</td>
<td>( \lambda_x/2 )</td>
<td>2.6737</td>
<td>2.6885</td>
<td>3.1416</td>
</tr>
<tr>
<td>3</td>
<td>( \lambda )</td>
<td>( \lambda_x/2 )</td>
<td>3.1511</td>
<td>2.6885</td>
<td>3.1416</td>
</tr>
<tr>
<td>4</td>
<td>( 3\lambda/2 )</td>
<td>( \lambda_x )</td>
<td>5.2437</td>
<td>5.2706</td>
<td>6.2832</td>
</tr>
<tr>
<td>5</td>
<td>( 3\lambda/2 )</td>
<td>( \lambda_x )</td>
<td>6.2862</td>
<td>5.2706</td>
<td>6.2832</td>
</tr>
</tbody>
</table>

**TABLE 1.** Resonant wave numbers for a coupled beam cavity

As can be seen from the table, many of the coupled modes appear in pairs. There is however a relative phase difference between the acoustic and structural motions in two such modes, and their displacements are either structurally or acoustically dominated.

## REFERENCES