A wave based prediction technique for vibro-acoustics:
comparison with finite element technique and experimental validation

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Abstract: A wave based prediction technique for steady-state vibro-acoustic analysis is being developed. The aim is to obtain a high prediction accuracy from substantially smaller models than required with the existing element based models, so that the technique can be applied up to much higher frequencies. In this paper, the technique is applied for the prediction of the pressure in the cavity of a car breadboard model by applying the measured structural velocity distribution as boundary conditions to the car cavity. The prediction results of the wave model are compared with the finite element predictions and the measured cavity pressure field.

INTRODUCTION

In the finite element method (FEM), the dynamic variables within each element are expressed in terms of nodal shape functions, which are usually low order polynomial functions. Since these functions are no local solutions of the governing dynamic equations, a substantial amount of elements is required in order to accurately represent the spatial variation of the dynamic response. This results in large models, whose size increases for increasing frequency. Since the subsequent computational effort increases also, the method can only be applied for a limited frequency range. In the wave based prediction technique, the domain is no longer divided into small elements. The dynamic variables in the entire domain, or at least in large subdomains, are expressed in terms of plane wave functions, which are exact solutions of the homogeneous part of the governing dynamic equations. To these wave functions, particular solutions of the inhomogeneous equations are added, so that the dynamic equations are exactly satisfied. The contributions of the wave functions to the dynamic response are determined by applying the boundary conditions in a weighted residual formulation. Since an approximation is induced only in the representation of the boundary conditions, a high accuracy is obtained from substantially smaller models, so that the technique can be applied up to much higher frequencies. A detailed discussion of the technique for coupled vibro-acoustic analysis is given in references (1) and (2).

WAVE MODEL

In this paper, the wave based prediction technique is applied for an uncoupled vibro-acoustic problem. A car breadboard model has been constructed, as shown in figure 1. It consists of a stiff beam structure (rigid body frame) and a subframe, which are connected through some rubber mounts. The engine excitation is simulated by exciting a mass with an electrodynamic shaker. The mass is isolated from the subframe by means of four rubber mounts. Aluminium panels of 3mm thickness are sealed to the beam structure with silicone to constitute the car cavity. The pressure in the cavity is simulated using the measured structural velocity of the aluminium panels. Since no acoustic sources are located in the cavity volume \( V \), the steady-state fluid pressure \( p(\mathbf{r}) \) at each point with co-ordinate \( \mathbf{r}(x,y,z) \) is governed by the homogeneous Helmholtz equation and the boundary conditions along the boundary surface \( \Gamma \) of the cavity,

\[
\nabla^2 p(\mathbf{r}) + k^2 p(\mathbf{r}) = 0, \quad \mathbf{r} \in V \quad \text{and} \quad -\frac{j}{\rho \omega} \frac{\partial p(\mathbf{r})}{\partial n} = v_n(\mathbf{r}), \quad \mathbf{r} \in \Gamma, \quad \tag{1,2}
\]

where \( k(=\omega/c) \) is the acoustic wavenumber, \( \omega \) is the circular frequency, \( \rho \) is the fluid density, \( c \) is the sound speed and \( v_n \) is the measured structural velocity component in the direction normal to the boundary surface.

An expression \( p \) in terms of acoustic plane wave functions \( \phi_a \) is proposed as solution for the pressure field \( p \),

\[
\bar{p}(x,y,z) = \sum_a P_a \phi_a(x,y,z) = \sum_a P_a e^{-j(k_{rx}x+k_{ry}y+k_{rz}z)}. \tag{3}
\]
The wavenumber components \((k_{x_1}, k_{y_1}, k_{z_1})\) of the wave functions are a truncated selection from

\[
\left(\frac{m_1 \pi}{L_x}, \frac{n_1 \pi}{L_y}, \sqrt{k^2 - \left(\frac{k_{x_1}}{L_x}\right)^2 - \left(\frac{k_{y_1}}{L_y}\right)^2}\right), \left(\frac{m_2 \pi}{L_x}, \frac{n_2 \pi}{L_y}, -k_{z_2}, \sqrt{k^2 - \left(\frac{k_{x_1}}{L_x}\right)^2 - \left(\frac{k_{y_1}}{L_y}\right)^2}\right), \text{ and } \left(\sqrt{k^2 - \left(\frac{k_{x_1}}{L_x}\right)^2 - \left(\frac{k_{y_1}}{L_y}\right)^2}, \frac{m_3 \pi}{L_y}, \frac{n_3 \pi}{L_z}\right),
\]

with \(m, n = 0, \pm 1, \pm 2, \ldots\) and where \(L_x, L_y, L_z\) are the dimensions of the smallest enclosing rectangular parallelepiped of the domain (see ref. 2). For the considered cavity, these dimensions are \(L_x = 1500\text{mm}, L_y = 975\text{mm}\), and \(L_z = 800\text{mm}\) (see figure 1). No matter what the values of the wave contributions \(P_i\) are, the Helmholtz equation is exactly satisfied. The wave contributions are determined from the orthogonalisation of the approximation error of the boundary conditions with respect to a weighting function \(w'_i\). Each wave function in the proposed solution \(\tilde{p}\) is used for the construction of the weighting function \((\cdot)^*\) (complex conjugate).

\[
\forall \phi_i: \int_{\Omega} w'_i(r) \left(\frac{j}{\rho_0} \frac{\partial \tilde{p}(r)}{\partial n} - \tilde{v}_i(r)\right) d\Omega = 0, \text{ with } w'_i(r) = \left(\frac{\partial \phi_i}{\partial n}\right)^*.
\]

**DISCUSSION OF RESULTS**

A finite element model of the cavity is built with 9600 linear hexahedral fluid elements and 11067 nodes, using the structural velocity input, measured with a laser vibrometer at each node location on the boundary surface. Figure 2 compares the measured cavity pressure field with the prediction results from the FE model and from a wave model with 384 wave functions. It illustrates that the wave method provides a high accuracy with a substantially smaller model compared with the FE method. Especially around the acoustic eigenfrequencies (see 214 Hz, 219 Hz and 243 Hz), where the response is very sensitive to the velocity input, the wave method yields a better accuracy. Compared with the FE method, the wave method yields also a reduction of the computational effort, but not to the same extent as the model size reduction, since an uncoupled FE model has a banded, symmetric structure while a wave model is fully populated and non-symmetric. For coupled vibro-acoustic problems, for which FE models are also non-symmetric, the computational savings of the wave method are more pronounced.

**FIGURE 1.** Car breadboard model (exploded view)  
**FIGURE 2.** Pressure at a cavity point and prediction error

**REFERENCES**

2. Desmet, W., Sas, P., and Vandepitte, D. “A wave based prediction technique for three-dimensional coupled vibro-acoustic analysis,” submitted for publication in *JASA*.