A Study of Reverberant Fields in Rooms with Various Absorptions - Including Prediction of Spatial Variation

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Abstract: A rectangular acoustic cavity with sound sources on one wall, an absorptive endwall, and two absorptive sidewalls is modeled. Using an intensity balance at cross-sectional areas, a differential equation for mean-square pressure in the acoustic cavity is developed. This leads to an approximate, simple formula for the mean-square pressure as a function of distance from source-wall to absorbing endwall. The other variables in the formula include, the absorption coefficients, cross-sectional area and perimeter, and source power. This formula is compared to previous results from numerical simulations, and to classical Sabine predictions for mean-square level.

INTRODUCTION

A common assumption in room acoustics is that the sound pressure levels in a reverberant field are uniform, that the sound field is diffuse, and that the flow of energy is equally probable in any direction. Based upon these assumptions, several simple theories have been developed to predict the overall sound pressure level in such spaces. The most basic of these predictions is the classical result of Sabine.

In this work, a new simple theory for predicting sound pressure levels in a moderately reverberant room is developed using conservation of energy principles. This result is then compared to previous results from a computer simulation of the sound field in a room with absorbing surfaces and a distribution of point sources on a wall. The new theory agrees well with the computer simulation, and gives approximately the same overall spatial average as the classical Sabine result.

THEORETICAL DEVELOPMENT

A rectangular room, with sound sources on one wall and absorbing surfaces on three other walls, is shown in Fig. 1.

FIGURE 1. Rectangular room, with source wall (x=Lx), absorbing endwall (x=0), and absorbing sidewalls (y=Lz and z=Lz).

The average intensity of oblique right-traveling (+x) waves at a cross-section of width Δx and area S, is

\[ I_x = \frac{\bar{p}_x^2}{2pc} \]

in the x direction. The corresponding intensity at the sidewalls is \( I_y = I_z = I_s = \frac{1}{2} \frac{\bar{p}_s^2}{2pc} \frac{2}{2-\alpha_w} \),

where \( \bar{p}_s^2 \) is the mean-square pressure of right-traveling waves, and \( \alpha_w \) is the absorption coefficient at the sidewalls.

Note, the intensity formula is not the standard one, due to the inclusion of right-traveling oblique waves only (this accounts for a factor of 2). At the sidewalls, there is a reduction of 1/2 due to the presence of the walls, and an additional factor due to the reduced pressure amplitude which is reflected.

Define \( \beta \) to be the fraction of surface area at a cross-section covered by absorbing material whose random incidence absorption coefficient is \( \alpha_w \); and define, \( L_p \) and \( S \) as the length of the perimeter of the cross-section and its area, respectively. An equation for the change in intensity across Δx can be written in terms of the intensity lost through the sidewalls, as:

\[ S \frac{dI_s}{dx} \Delta x = -\alpha_w \beta L_p \Delta x I_s. \]

Rewritten, in terms of mean-square pressure:
\[
\frac{d(p^2)}{dx^2} + \frac{\alpha_x \beta_x}{(2 - \alpha_x)S} p^2 = 0.
\]

Solves this differential equation. Similarly, for left-traveling oblique waves, \( p^2 = P e^{\frac{-\alpha_x \beta_x}{(2 - \alpha_x)S} x} \). At the absorbing endwall (x=0), the intensity of the left-traveling waves equals the reflection coefficient times the intensity of the right-traveling waves. In other words, \( I_{ul}(0) = (1 - \alpha_b) I_{ur}(0) \) and therefore, \( P_I = (1 - \alpha_b) P_r \), where \( \alpha_b \) is the random incident absorption coefficient for the absorbing endwall. Assuming the left and right-traveling fields are uncorrelated, the net mean-square pressure is just a superposition of \( p_{ul}^2 \) and \( p_{ur}^2 \). At the source wall, \( x = -L_x \), the power of the source, \( W_s \), equals the cross-sectional area, \( S \) times the difference between the intensities, \( I_{ul} \) and \( I_{ur} \). From this relationship, the mean-square pressure as a function of the distance (x) from the source-wall to the end-wall can be written in terms of source power, absorption coefficients, and various dimensions of the room:

\[
\frac{p^2}{S} = \frac{2 \rho c W}{S} \left[ \frac{\cosh \left( \frac{\alpha_x \eta L_p}{(2 - \alpha_x)S} x \right) - \frac{1}{2} \alpha_b e^{\frac{-\alpha_x \eta L_p}{(2 - \alpha_x)S} x}}{\sinh \left( \frac{\alpha_x \eta L_p}{(2 - \alpha_x)S} L_x \right) + \frac{1}{2} \alpha_b e^{-\frac{-\alpha_x \eta L_p}{(2 - \alpha_x)S} L_x} \right]
\]

This result is compared to numerical simulations, in the next section.

**RESULTS**

The theoretical curves have been compared to exact results from previous computer simulations.\(^1\) A typical comparison is shown in Figure 2. This is a plot of reverberant pressure versus distance (1 = source wall, 0 = absorbing wall). The source power used in both the Sabine prediction and the theoretical formula above have been reduced to account for that which is absorbed on the first reflection. This gives a more accurate prediction of the reverberant sound field. The case shown is for 18.8 English sabines, for a room that is 2\'x3\'x7\' with all three absorbing walls equally absorbent. Other cases, including those where the absorption is not equally distributed, also show excellent agreement. Note the Sabine prediction will always yield a constant value in the room, and is not capable of predicting the spatial variation. This simple theory does a very good job of approximating this variation.

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**REFERENCES**