Model Experiments and Predictions of Sound Field in a Downward Refracting Medium

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Abstract Laboratory experiments are conducted to investigate the diffraction of sound by concave surfaces for both monopole and dipole sources. The cylindrical concave surface in an otherwise homogeneous medium is used to simulate an outdoor situation corresponding to a downward refracting medium. Normal mode solutions for monopole and dipole sources are developed that are based on a conformal transformation in which a stricter analogy is established.

INTRODUCTION

Diffraction of waves by convex surfaces to simulate sound propagation in an upward refracting medium has been studied in great detail. However, there is relatively little attention focused on the diffraction of waves by concave surfaces to simulate sound propagation in a downward refracting medium and most of these earlier studies have been concerned with monopole sources. In this presentation, we study sound propagation due to monopole and dipole sources in a downward refracting medium by examining the sound field above a concave surface both theoretically and experimentally.

NORMAL MODE SOLUTIONS FOR MONOPOLE AND DIPOLE SOURCES

It has been suggested that a strict acoustic analogy exists between downwardly curving ray paths over a plane boundary with an exponential profile, rather than a bilinear profile, of sound speed and a straight line sound propagation above a cylindrical concave surface in a neutral atmosphere. The sound speed is expressed in terms of the vertical height z as \( c(z) = c(0) \exp(z/R_c) \), where \( R_c \) is the radius of the concave surface. We note that \( R_c^{-1} \) may be interpreted as the normalized sound speed gradient for the downward refracting atmosphere. The normal mode solution for the sound field due to a monopole source can be derived to yield (1,2)

\[
p = e^{i\omega t} \sqrt{\frac{8\pi}{\nu}} \sum_n \left[ \frac{\xi_n \bar{\xi}_n}{k^2(z) m_n^2} \right] \frac{1}{4} \frac{\sqrt{K_n \alpha_i(-\xi_n)} \alpha_i(-\xi_n) e^{i\nu x}}{\frac{\partial^2}{\partial K_n^2} \left[ \alpha_i^2(\tau_n) - \left[ \alpha_i'(\tau_n) \right]^2 + \frac{\partial \alpha_i}{\partial K_n} \right]}
\]

\[
\xi(z) = \begin{cases} 
\frac{3}{2} K_n R_c \cos \psi, & \text{if } z < \text{Re}(z_1) \\
- \frac{3}{2} K_n R_c \cos \phi, & \text{if } z > \text{Re}(z_1)
\end{cases}
\]

\[
z_i = R_c \ln \left( \frac{\sqrt{k_0^2 - K_n^2 \sin^2 \psi}}{K_n \cos \psi} \right), \quad \frac{\partial \tau_n}{\partial K_n} = \frac{R_c}{\sqrt{-\tau_n}} \sec \psi, \tan^{-1} \left( \frac{\sqrt{k_0^2 - K_n^2}}{K_n \cos \psi} \right)
\]

\[
q_n = i k_0 \beta \sqrt{\tau_n / (K_n^2 - k_0^2)}, \quad \bar{k}_n(z) = + \sqrt{k_0^2 - K_n^2 \sin^2 \psi} \exp(-2z/R_c) \frac{K_n^2 \cos^2 \psi}{K_n \cos \psi}.
\]
In the above, \( \psi_r \) is the azimuthal angle of the receiver in the plane of constant \( z \), \( \tau_n = -\xi(0) \) are the zeros of \( \text{Ai}'(\tau_n) + q_n(\tau_n) = 0 \) and \( \beta \) is the normalized specific admittance of the impedance surface. The numerical values of \( \tau_n, q_n \) and \( K_n \) can be determined by using the method described in Refs. (1). Noting that the horizontal range and the vertical height dependent factors are not coupled in the normal mode solution for a monopole source, the dipole field \( p_d = p_h + p_v \), where \( p_h \) and \( p_v \) are the horizontal and vertical components of a dipole respectively, can be derived from the monopole field and written as

\[
p_d = e^{i\alpha} \sin \gamma \cos(\psi_d - \psi_r) \sqrt{\frac{2\pi}{r}} \sum_n \frac{\xi^2(\tau_n) \xi(\tau_n)}{k^2(\tau_n) k^2(\xi)} \left( 2iK_n + 1 \right) \frac{\partial \tau_n}{\partial k} \left[ \text{Ai}(\tau_n)^2 - \text{Ai}'(\tau_n)^2 \right] + \frac{\partial q_n}{\partial k} \left[ \text{Ai}(\tau_n)^2 \right],
\]

\[\text{(5)}\]

**EXPERIMENTAL RESULTS AND CONCLUSIONS**

Figure 1 shows the experimental results obtained at a frequency of 4350 Hz over a felt-covered cylindrical concave surface from monopole, horizontal and vertical dipole sources respectively. For comparison, the normal mode predictions for a bilinear profile (3) are also presented by the dotted curves in these figures.

Good agreement has been found between measurements and normal mode solution using an exponential sound speed profile. However, the agreement is less satisfactory where the sound field is due to a vertical dipole source.

**FIGURE 1.** Transmission loss due to (a) monopole, (b) horizontal dipole and (c) vertical dipole sources obtained over a felt-covered concave surface with \( R = 2.5 \) m, \( z = z = 0.1 \) m. Solid curves: the normal mode predictions by Eqs. (1) and (5), dotted curves: the normal mode predictions for a bilinear profile, circles: measured data.

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**REFERENCES**