Visualization of the Energy Flux in an Ensonified Fluid Loaded Elastic Sphere

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Abstract: The energy flux in an ensonified fluid loaded elastic sphere is visualized by its Poynting vector field in both its instantaneous real and time-averaged complex forms. The instantaneous Poynting vector field is shown in both freeze-frame and animated versions. The instantaneous Poynting vector field can be used to follow the propagation of elastodynamic waves in the elastic sphere. The complex Poynting vector field, which is a time-averaged representation of the energy flow, is shown in terms of separate real and imaginary parts. The real part of the complex Poynting vector field shows the traveling wave component of the elastodynamic wave in the ensonified sphere. The imaginary part of the complex Poynting vector field shows the standing wave part of the elastodynamic wave in the ensonified sphere. The patterns formed by the Poynting vector field, in both its instantaneous and the real and imaginary parts of the complex form, can be identified with the various modes of the ensonified sphere and used to explain the physical processes going on inside the ensonified sphere.

THE ENERGY FLUX IN AN ELASTIC MEDIUM

The energy flux in an elastic medium can be displayed by means of the elastodynamic Poynting vector. Most discussions of the Poynting vector center around the acoustic case and are usually limited to the time-averaged energy flux. The time averaged Poynting vector has its uses, particularly in its complex form, as explained below. The instantaneous Poynting vector is [see Auld(1)]:

\[ \vec{\mathcal{S}}_{\text{inst}} = -\vec{v} \cdot \vec{T}, \]

where \( \vec{T} \) is the stress tensor for an isotropic, lossless, elastic medium, and \( \vec{v} \) is the particle velocity at a given point in the medium. Assuming simple harmonic time variation for all appropriate physical variables of the form \( e^{i(kr - \omega t)} \), where \( k \) is the wavenumber and \( \omega \) is the angular frequency of the wave, the complex time-averaged Poynting vector for an elastic medium can be derived as

\[ \vec{\mathcal{S}}_c = -\frac{i}{4} \vec{v}^* \cdot \vec{T}, \]

where \( \vec{v}^* \) is the complex conjugate of the now complex particle velocity, and where \( \vec{T} \) is the now complex stress tensor for an isotropic, lossless, elastic medium.

The instantaneous Poynting vector not only allows inspection of the time evolution of the energy flux vector field over a cycle of the sound wave, but also has the interesting geometric property of tracing an ellipse, twice per cycle, in a manner and with axes and orientation that are easily defined, as was shown by Dean and Braselton(2) in a previous paper. The complex Poynting vector, by way of contrast, shows the time-averaged behavior of the elastodynamic wave in two distinct ways. The real part of the complex elastodynamic Poynting vector field is the time average of the instantaneous elastodynamic Poynting vector field and can be used to visualize traveling wave patterns in the elastic medium. The imaginary part of the complex elastodynamic Poynting vector field will vanish for the case of a simple plane wave, and its presence signals the existence and shows the geometric form of a standing wave in the medium.

THE FLUID LOADED ENSONIFIED ELASTIC SPHERE

The case of a fluid loaded ensonified elastic sphere will be examined in terms of time animated plots of the instantaneous elastic Poynting vector field, and both the real and imaginary parts of the complex elastic Poynting vector field. The plots will be displayed both to bring out important details of the energy flow at particular values of the size parameter (defined with respect to the fluid medium) and to show important aspects of both the time evolution of the instantaneous vector field and the evolution with size parameter of both the real and imaginary parts of the complex vector field.
As an example of the kind of plot to be shown, here is a typical example with Lamé coefficients typical of yellow brass, surrounded by water, with a size parameter of 2.68 relative to the incident compressional wave. The particle velocities and stress tensor elements are solved for from first principles. Then since the Poynting vector field has axial symmetry, only one half of one longitudinal slice is shown. The maximum Poynting vector has been scaled to just equal the radial distance between adjacent field points. There is clear indication of several radial traveling wave lobes, further complicated by sudden reversals in a given lobe from radially inward to radially outward energy flow when crossing a particular radial distance.

FIGURE 1. Time averaged real part of the complex elastic Poynting vector field for a brass sphere in water with \( ka = 2.68 \).

REFERENCES