Parametric Efficiency of a
Bi-frequency Focusing Gaussian Nonlinear Source

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Abstract: Difference frequency sound distribution from a bi-frequency focusing nonlinear source was calculated. The simple formulation of Novikov et al. (1) for the Gaussian wavefront was adopted for simulations. Radius of aperture or beamwidth \(a\), radius of curvature \(D\), mean primary frequency \(f_m\), and difference-frequency \(f_d\) are essential parameters describing the performance of a parametric focusing source. The geometrical ratio \(D/a\), and the down-shift ratio \(f_m/f_d\) were used as control parameters for the investigation of the source's performance. The parametric efficiency is defined for the bi-frequency focusing source and plotted for different values of \(D/a\) and \(f_m/f_d\).

PROBLEM STATEMENT

Sound propagation from a bi-frequency focusing Gaussian nonlinear source is investigated in details by various research groups (1,2). In another study (3), it was also shown that difference frequency wave radiation from a diffractive focusing source can be modeled with an ideal Gaussian source having parameters optimized according to the primary wave intensity. As a consequence of using Gaussian amplitude shading, closed form analytic solutions are obtained for primary and difference frequency wave distributions. Propagation process is considered to be in a homogeneous and lossless water medium.

The focusing source has a radius of \(a\), and a radius of curvature \(D\). \(f_1\) and \(f_2\) are the operating frequencies of the bi-frequency source. Defining \(f_m = (f_1 + f_2)/2\), the down-shift ratio \(f_m/f_d\) becomes an important parameter for the performance of the bi-frequency source, where \(f_d = f_1 - f_2\) is the difference-frequency. The geometrical ratio \(D/a\) of the source is another critical parameter for the nonlinear sound generation.

The difference frequency pressure distribution due to a bi-frequency primary wave with a Gaussian amplitude shading at the source is

\[
p_d(z, r) = i\kappa \left[ \frac{2\pi f_d p_0_1 p_0_2 R_d}{c_0^2 \rho_0} \int_0^{Z_d} \exp \left( \frac{2z_d^2}{R_d} \left[ 1 + \frac{2m \kappa}{R_d} + i \kappa \delta \left( 1 - \frac{R_d}{z_d} \right) \right] \right) \right] \frac{B_m(y)}{B_m(y)} dy. \tag{1}
\]

Here \(\kappa\) is the parameter of nonlinearity, \(c_0\) is the velocity of sound, \(\rho_0\) is the density of the medium, and \(p_0_1\) and \(p_0_2\) are the peak source pressures of the primary waves. All of the lengths are normalized according to the Rayleigh distance \(R_d\) of the difference frequency sound, i.e. \(z_d = z/R_d, D_d = D/R_d,\) and \(B_m(y) = [(1 - y/D_d)(1 - z_d/D_d) + i(y - z_d) + 2imy(1 - z_d/D_d) + m^2 z_d]. m = f_2^2/4f_1f_2\) is a parameter defining the mutual interaction of primary and difference frequency waves. \((m < 10^{-2}\) in practice.)

The axial distribution of the difference frequency sound can be found from Equation (1) with \(r = 0\) as

\[
p_d(z, 0) = \kappa i [z_d^2 + m z_d^2 + 2imz_d] \left[ \frac{m z_d - z_d/D_d + i(1 + 2mz_d)}{m z_d - z_d/D_d + i(1 + 2mz_d)} \right]^{1}, \tag{2}
\]

where \(z_d = (1 - z_d/D_d),\) and \(\kappa = \beta \pi \rho_0 p_0_1 p_0_2 R_d / (c_0^2 \rho_0).\) (Refer to (1) for details of the derivation.)

In this study, practical values from biomedical applications are selected for source parameters and the efficiency of difference frequency sound generation is investigated for various \(D/a\) and \(f_m/f_d\) values. [I] For \(f_m = 1\) MHz, and \(f_d = 50\) KHz \((f_m/f_d = 20)\) difference frequency sound generation is calculated for \(D/a\) values between 1 and 20. This is conducted in two parts: (i) \(a\) is fixed to 2.4 cm and \(D\) is adjusted, and (ii) \(D = 4.8\) cm is selected and \(a\) is varied accordingly. [II] Similarly, for \(a = 2.4\) cm and \(D = 4.8\) cm (i.e. \(D/a = 2)\) difference frequency sound distribution is obtained for \(f_m/f_d\) beginning from 10 up to 100, in two parts: (i) \(f_m\) is fixed to 1 MHz and \(f_d\) is varied, and (ii) \(f_d = 50\) KHz is selected where \(f_m\) value is changed.

Parametric efficiency of the bi-frequency source is defined as \(\eta(r, z) = [p_d(r, z)] / [P_1(r, z) P_2^2(r, z)],\) where \(P_j (j = 1, 2)\) is the primary wave pressure at frequency \(f_j.\) Since we are interested in the axial propagation
of the waves, $\eta(r,0)$ is plotted against $z/D$.

RESULTS

Results of each case are given below. It is observed in Figure (1) that $\eta(z/D)$ makes a minimum just before the geometric focus. As $D/a$ increases ($D$ increases), this minimum slightly moves towards the source and it disappears for $D/a > 10$. As $D \to \infty$, the source behaves like an unfocused (planar) transducer. The maximum point before the focus has a similar behaviour: it moves towards the source as $D/a$ increases and then disappears. (A more exact limit is $D/a = 16$.)

Figure (2) demonstrates that sharp occurrence of the minimum and maximum points in the prefocal region is dominant for values of $D/a < 5$. As $D/a$ increases ($a$ decreases), these minimax points disappear and the focusing behaviour becomes eliminated. This is expected, since for $a \to 0$, the source behaves like a point source.

Figure (3) tells us that location of the minimum point does not affected by the down-shift ratio. The maximum efficiency in the prefocal region is a function of $f_m/f_d$, on the other hand. As $f_m/f_d$ decreases ($f_d$ increases), this maximum point moves towards the source. Also, we observe that the larger the down-shift ratio, the smaller the $\eta(z/D)$, along the $z$-axis.

Although the last plot in Figure (4) shows that the axial distribution of $\eta$ is mainly the same for the frequencies of interest, this is not the situation for larger values of $D$. The inverse proportionality between $\eta$ and $f_m/f_d$ still holds in this case.

REFERENCES