An analytical model for band-limited response of vibroacoustic systems

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Abstract: An analytical method for predicting frequency band-limited time- and space-averaged sound pressure in vibroacoustic systems is presented. Cases of structure-borne sound are considered where an enclosed sound field is subject to acoustic-structural coupling with various vibrating flexible boundary structures. This method can be useful for medium frequency analysis where there is a large number of dominant acoustic and/or structural modes. It reduces computational effort in summing the modal contribution of each subsystem mode given the information of the modes. In this paper, the prediction accuracy of the present method will be compared to the classical modal coupling method and its computational efficiency is demonstrated by mean of numerical examples.

BAND-LIMITED SOUND FIELD RESPONSE

The band-limited method described in this paper is based on the classical modal coupling method and a number of approximations have been applied in the mathematical formulation of an analytical expression for the enclosed sound field response. These approximations are described and justified in Ref. 1. A summary of results are presented here for plate-type boundary structures and cases of direct excitation of one structure are considered. The frequency band-limited time- and space-averaged mean-square sound field pressure is given by

\[
\langle p^2 \rangle_{\text{a,s}} = \frac{\rho_v^3 c_0^4}{2V_0 \Delta \omega} \sum_{i=1}^{N} \frac{1}{M_i} \sum_{j=1}^{M_i} \left[ B_i^E M_i^E \right] \left[ \int_{A_i^E} P_{\text{ext}} S_j \, d\sigma \right] \left[ \int_{A_i^E} p_{\text{ext}}^* S_j \, d\sigma \right] \left[ I_i(\omega_U) - I_i(\omega_L) + I_2(\omega_U) - I_2(\omega_L) \right]
\]

where

\[
I_i(\omega) = \frac{(c_{i,j} A_{i,j} + d_{i,j} B_{i,j})}{2(A_{i,j}^2 + B_{i,j}^2)} \ln \left( \frac{(\omega + A_{i,j}^2 + B_{i,j}^2)}{(\omega - A_{i,j}^2 + B_{i,j}^2)} \right) + \frac{(c_{i,j} B_{i,j} - d_{i,j} A_{i,j})}{2(A_{i,j}^2 + B_{i,j}^2)} \tan^{-1} \left( \frac{4B_{i,j} \omega(\omega^2 - A_{i,j}^2 - B_{i,j}^2)}{(\omega^2 - A_{i,j}^2 - B_{i,j}^2)^2 - (2B_{i,j} \omega)^2} \right),
\]

\[
I_2(\omega) = \frac{(c_{i,j} C_{i,j} + f_{i,j} D_{i,j})}{2(C_{i,j}^2 + D_{i,j}^2)} \ln \left( \frac{(\omega + C_{i,j}^2 + D_{i,j}^2)}{(\omega - C_{i,j}^2 + D_{i,j}^2)} \right) + \frac{(c_{i,j} D_{i,j} - f_{i,j} C_{i,j})}{2(C_{i,j}^2 + D_{i,j}^2)} \tan^{-1} \left( \frac{4D_{i,j} \omega(\omega^2 - C_{i,j}^2 - D_{i,j}^2)}{(\omega^2 - C_{i,j}^2 - D_{i,j}^2)^2 - (2D_{i,j} \omega)^2} \right),
\]

\[
A_{i,j} = \omega^E_j \left( \sqrt{1 + \frac{\eta_{i,j}^E}{\omega_j^E}} \right)/2,
\]

\[
B_{i,j} = \omega^E_j \left( \sqrt{1 + \frac{\eta_{i,j}^E}{\omega_j^E}} - 1 \right)/2,
\]

\[
C_{i,j} = \omega_{m,i} \left( \sqrt{1 + \frac{\eta_{i,j}^m}{\omega_{m,i}}} \right)/2,
\]

\[
D_{i,j} = \omega_{m,i} \left( \sqrt{1 + \frac{\eta_{i,j}^m}{\omega_{m,i}}} - 1 \right)/2,
\]

\[
c_{j,j} = -c_{i,j},
\]

\[
f_{j,j} = \eta_{i,j}^E \left( \omega^E_j \left( \sqrt{1 + \frac{\eta_{i,j}^E}{\omega_j^E}} ^2 + \omega_j^E \left( \omega_j^E \left( \sqrt{1 + \frac{\eta_{i,j}^E}{\omega_j^E}} - 1 \right) \right) \right) - 2\omega_j^E \left( \sqrt{1 + \frac{\eta_{i,j}^E}{\omega_j^E}} \left( \sqrt{1 + \frac{\eta_{i,j}^E}{\omega_j^E}} - 1 \right) \right) \right)/g_{j,j},
\]

\[
g_{j,j} = 2\eta_{i,j}^E \left( \omega^E_j \left( \sqrt{1 + \frac{\eta_{i,j}^E}{\omega_j^E}} ^2 + \omega_j^E \left( \omega_j^E \left( \sqrt{1 + \frac{\eta_{i,j}^E}{\omega_j^E}} - 1 \right) \right) \right) - 2\omega_j^E \left( \sqrt{1 + \frac{\eta_{i,j}^E}{\omega_j^E}} \left( \sqrt{1 + \frac{\eta_{i,j}^E}{\omega_j^E}} - 1 \right) \right) \right)/g_{j,j},
\]

\[
\theta^E_j = \eta_{i,j}^E + \left( \frac{1}{M_{i,j}^E} \frac{\eta_{i,j}^E}{\omega_j^E} \right) \frac{2}{\omega_j^E} \sum_{m=1}^{M_{i,j}^E} \sum_{j=1}^{M_{i,j}^E} \left( A_{i,j}^E B_{i,j}^E \omega_j^E \right)^2 \left[ I(\omega_U) - I(\omega_L) \right],
\]

\[
I(\omega) = \frac{(c_{i,j} F_{i,j} + d_{i,j} G_{i,j})}{2(F_{i,j}^2 + G_{i,j}^2)} \ln \left( \frac{(\omega + G_{i,j}^2 + F_{i,j}^2)}{(\omega - G_{i,j}^2 + F_{i,j}^2)} \right) + \frac{(c_{i,j} G_{i,j} - d_{i,j} F_{i,j})}{2(F_{i,j}^2 + G_{i,j}^2)} \tan^{-1} \left( \frac{4G_{i,j} \omega(\omega^2 - F_{i,j}^2 - G_{i,j}^2)}{(\omega^2 - F_{i,j}^2 - G_{i,j}^2)^2 - (2G_{i,j} \omega)^2} \right),
\]

\[
E_{i,j} = \omega_j^E \left( \sqrt{1 + \frac{\eta_{i,j}^E}{\omega_j^E}} \right)/2,
\]

\[
c_{i,j} = -0.5,
\]

\[
d_{i,j} = 0.5/\eta_{i,j}^m.
\]
$M_i$, $M_s$, $\omega_i$, $\omega_s$ are acoustic and structural modal masses and resonance frequencies. $\eta_{ij}$ and $S_j$ are structural modal loss factor and mode shape function. Quantities with superscript "E" belong to the structure which is directly excited. $\bar{\eta}_{iu}$ and $\bar{\eta}^c_{iu}$ are the band-averaged uncoupled and coupled acoustic modal loss factors, $N_\tau$ is the number of vibrating structures, $p_{\text{ext}}$ is the external sound pressure, $A_i$ is structural surface area, $B_{ji}$ is the modal coupling coefficient, $c_0$ is speed of sound in air, $\rho_0$ is air density and $V_0$ is the volume of the enclosure. $\alpha_i$ and $\alpha_s$ are the upper and lower limits of a given frequency band. Values of the above inverse tangents vary from 0 to $2\pi$.

RESULTS AND DISCUSSION

Numerical examples are presented for a system which consists of a rectangular parallelepiped acoustic enclosure of dimensions $(0.868, 1.150, 1.000)$ m with two rectangular simply-supported aluminium plates. Plate 1 (thickness, $h_1=3$ mm) is located at $z=1.000$ m and plate 2 (thickness, $h_2=1$ mm) is located at $z=0.000$m. Each plate is sequentially excited by a steady-state mechanical point force which is randomly positioned. Narrow excitation bandwidths ($\Delta\omega=9\times2\pi$ rad/s) are used. Approximated SPL calculated by the present method and exact SPL obtained from the classical modal coupling method are compared. Figure 1 shows that the results at medium frequencies (eg. $>500$ Hz) are in good agreements. Figure 2 shows that only a small percentage of actual calculation time is required if the present method is used for prediction of band-limited SPL since inversions of large complex matrices as in the classical modal coupling method have been avoided. Both cases where the sound field is coupled to one plate and to two plates, are shown. Calculations at each single frequency and subsequent frequency integration for band-limited SPL have also been avoided. The SPL in a given band can be predicted straight away in one calculation. Thus, the computational time improves as $\Delta\omega$ increases (see Fig. 2b).

**FIGURE 1.** SPL for $\Delta\omega=9\times2\pi$ rad/s when (a) only plate 1 ($h_1=3$ mm), and (b) only plate 2 ($h_2=1$ mm), is directly excited.

**FIGURE 2.** Percentage of actual calculation time as a function of (a) averaged modal density of plates, for $\Delta\omega=9\times2\pi$ rad/s, and (b) $\Delta\omega$, for averaged modal density of plates of $0.1679$ Hz$^{-1}$.

REFERENCES