Time Variation Characterization of a Non-stationary Time Series

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Abstract: The Fourier transform of a non-stationary time series has the difficulty of providing a proper physical interpretation of the signal as the physical attributes and dynamics associated with the time series may consist of several transients that are different in their nature. The time series analysis method originated by N. Huang [1] has a unique approach to adaptively break down the non-stationary time series into the sum of several simple mode functions. Each single mode function can be expressed in the form of an analytical signal, whose amplitude and phase can both be varied with time. Those time variations can be related to the abrupt changes associated with dynamic systems to observe the occurrence of a transient event in a non-stationary process.

INTRODUCTION

Signal analysis based on the Fourier transform is to express a time series in terms of a combination of harmonic functions, sine and cosine functions. For a non-stationary signal, the frequency components so obtained from the harmonic functions only show the periodicity of the given time series and do not provide the dynamic nature of transient like behavior. Although the short time Fourier transform has been employed to alleviate such a problem, the limitation imposed by the ambiguity of time and frequency commutation problem has constrained its applications. The meaningful interpretation of the processed results very much depends on the time duration and the bandwidth of the given signal under investigation and the stationarity property associated with the physical phenomenon.

Recently the much publicized wavelet theory was suggested a new approach to decompose a signal in terms of a set of special base functions other than sine and cosine functions through a combination of delay and dilatation of a selected mother wavelet function. The wavelet transformation method, not requiring the stationary assumption, is more suitable to a short duration transient like signal. However there is no guideline regarding how to choose the mother wavelet and their results do not provide the association of the dynamic nature of the non-stationary signal.

ANALYTIC SIGNAL REPRESENTATION

A simple harmonic signal with a constant amplitude can be represented by a cosine/sine function with a fixed magnitude of $A$, a phase angle of $\phi$, which has a steady radian frequency of $\omega_0$:

$$s(t) = A \cos (\omega_0 t + \phi), \quad \text{with} \quad \phi_t = \omega_0 t + \phi, \quad \text{and} \quad \omega_0 = \frac{d\phi}{dt} = \dot{\phi}(t).$$

or in the form of a complex valued function:

$$s(t) = A \exp[i \phi(t)], \quad \text{where} \quad s(t) = \Re[s(t)], \quad \text{and} \quad \omega_o = \Im[\dot{s}(t) / s(t)].$$

Hence, following the same reasoning, a time varying non-stationary signal can be put into a similar form by extending this simple formulation, so-called the analytic signal format:

$$s(t) = a(t) \exp[i \phi(t)] = s(t) + j \nu(t),$$

where $\nu(t)$ is the Hilbert transform of $s(t)$. Its corresponding amplitude 'a' and radian frequency 'o' become, in this case, time varying functions and have the following relationships:

$$a^2(t) = s^2(t) + \nu^2(t), \quad \text{and} \quad \omega(t) = \Im[\dot{s}(t) / s(t)].$$

However, this simple formulation of expressing a time varying non-stationary signal in terms of time varying amplitude and time varying phase only makes meaningful association with a single mode function, which will be defined later. For a composite signal which is attributed from many different kinds of origination, the interferences among the different causes make the computed results for the time varying amplitude and phase, according to equation (3), ill-behaved and the original concept in connection with the traditional definition of amplitude and phase is not well matched.
TIME VARYING SIGNAL DECOMPOSITION

In order to resolve the composite time varying non-stationary signal (a multimode function) decomposition problem and to still follow the traditional concept of amplitude and phase, Huang [1] originated an innovative methodology based on Hilbert transform, the Hilbert-Huang transform (H^2T). First he gave a concise definition of single mode function as an intrinsic mode function whose properties are: (i) in the whole data set of the time series, the number of extrema and the number of zeros-crossings must equal or differ at most by one, (ii) at any point, the mean value of the envelope defined by the local maxima and envelope defined by local minima is zero. Then decomposition procedure can be carried out as the following:

(a) the time varying non-stationary signal is expanded in terms of the combination of intrinsic mode function, (b) each intrinsic mode function has a well defined time frequency character through analytic signal representation and Hilbert transform.

The processing algorithm can be summarized in following major steps:

1. \( m_i(t) = \frac{\text{avg}[s(t)_{\text{max}}] + \text{avg}[s(t)_{\text{min}}]}{2} \),
2. \( h_i(t) = x(t) - m_i(t) \),
3. \( m_{ik}(t) = \frac{\text{avg}[h_i(t)_{\text{max}}] + \text{avg}[h_i(t)_{\text{min}}]}{2} \),
4. \( h_{ik}(t) = h_{ik}(t_{-1}) - m_{ik}(t) \),
5. \( C_i(t) = h_{ik}(t) \),
6. \( s(t) = \sum C_i(t) + r_n(t) \),
7. By Hilbert Transform:

\( S(a, \phi)(t) = \sum C_i(a, \phi)(t) + r_n(a, \phi)(t) \)

APPLICATIONS

Figure 1 are the computation results from \( H^2T \) for a human voice of "CUP". The top left time series plot is the original recorded signature and its four major components, computed from \( H^2T \), are listed below consequentially at same column. The spectrogram in a joint time and frequency display is shown at the top right graph (t-f PLOT). The magnitude (supposed to be in color coded) and frequency's time variation of four major components is displayed as four distinct traces (VEIN diagram) in the time-frequency frame at lower right corner. They are consistent with each other in presenting the features of the speech. However, in the VEIN diagram, there are two well defined segments of chirp-like signal occurred around 0.014 sec (of components 2 and 3) which were not so obvious in the original signature and not well defined neither in the spectrogram. Such separable capability is useful for detecting the occurrence of certain characteristic transients in a noisy environment without implying any statistic assumption [2]. Potential utilization of \( H^2T \) in the detection of intrusion signals in underwater applications related to certain propagation modes or special features due to the response of a submerged structure warrants further investigation [3].

REFERENCES