Mathematical Model of Echolocation of Fish-Eating Bats

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Abstract: The problem of reflection of the spherical acoustical wave from the water with the spherical air inclusion under the surface was solved numerically. The frequency dependence of the pressure in reflected wave shows the significant attenuation at some frequencies. It gives the possibility to explain how the echolocating bat can detect fish under the water surface.

INTRODUCTION

Echolocating bats use sounds to mediate spatial perception. They transmit sonar signals and perceive the nature and location of objects from the echoes of these sounds that return to their ears (1,2). However, the ability of echolocating bats of the species Vespertilionidae to detect fish under the water surface using its own sonar is surprising, because only the one thousandth of the acoustical pulse energy penetrates into water. Supposing the fish (in fact its swimbladder) reflects the whole signal backwards, the bat must detect the echo that is the million times less than the emitted signal. This fantastic level of the perception sensitivity of the echolocating bat lies on the bound of its ability. It seems incredible, that the main method of getting food is provided for the maximum effort of bat senses. The purpose of this article is to derive the reasonable model of the bat's echolocation.

PROBLEM ANALYSIS

At first, it seems the swimbladder reflects the incident wave at the resonant frequency rather well, and this fact can help the bat to detect the fish. The usual range of echolocation of bat of the species Vespertilionidae lies in range 50 - 100 kHz. Because the resonance frequency of the swimbladder of the radius $a = 1 \text{ cm}$ is equal to $1 \text{ kHz}$ approximately, the swimbladder oscillations are not needed to the bat.

The other plausible explanation of how the echolocating bat detects fish is based on the resonant features of the water layer between the spherical air inclusion and the water surface. In this case we may see the phenomenon similar to the phenomenon of the plane wave propagation through the plane layer with the acoustical properties that are strongly different from the acoustical features of the medium in which the wave propagates. It is well known, that the acoustical wave reflects from this layer almost entirely backwards, but if the layer thickness is equal to the integer number of the half of the wave-length, then the layer is completely transparent. It is very interesting to make known does the similar phenomenon takes place in case of a one plane and another a spherical surface.

To verify this supposition the stationary problem of diffraction of a spherical acoustical wave by the water surface with the spherical air inclusion under the surface was formulated and solved. The pressure in reflected wave $P$ and in the same way the pressures of waves in water and inside the inclusion are presented in the form of the spherical function series. The constant coefficients of these expansions were find from the infinite system of algebraic equations. The approximate solution for the reflected wave pressure was obtained for the source point $H=6 \text{ cm}$ above the water surface and the spherical inclusion with radius $a=1 \text{ cm}$ submerged at the depth $h = 5 \text{ cm}$ for the frequency range $0.05 < k a < 20$. Here $k a$ is the nondimensional wave number in air. This nondimensional frequency range corresponds to the range $250 \text{ Hz} - 100 \text{ kHz}$.

The frequency dependence of pressure in reflected wave is shown in Fig. 1. For the high frequencies, if the bat detects the direct echo from the swimbladder we must see the flat line that is a constant value of the pressure amplitude with a very small ($0.000001$) modulation on it, and nothing else. However, we find that the amplitude of the wave reflected by the water surface depends on the frequency significantly. At some frequencies we may see the dips up to $25$% of the amplitude. The width of these dips is equal to $30 \text{ Hz}$ ($0.006$ in $k a$ units). Although this value is very small, the bat can detect the air inclusion in water by using frequency-modulated pulses more easily. Perhaps, the influence of the fish tissue surrounding the swimbladder expands these regions.
In fact, the echolocating bats of the species *Vespertilionidae* emit frequency-modulated pulses with the linear law of changing of frequency in time. The velocity of this alteration is equal to $100 \text{ kHz/s}$ approximately. So, it needs $0.3 \text{ ms}$ to change the frequency to $30 \text{ Hz}$. For this time in water layer of thickness $3 \text{ cm}$ maybe occur 15 reflections of wave. Thus, the process of the frequency changing is really slow enough, and we may consider the process of the echolocation as a stationary problem.

![Graph](image)

**FIGURE 1.** Frequency dependence of pressure in reflected wave.

It is easy to see, if the frequency of the emitted signal is equal to the frequency at which we have the dip, the echo from the water drops out significantly. This fact indicates the presence of the fish under the water surface.

**CONCLUSIONS**

Although the analysis in this article deals with the ideal model of the bat and swimbladder, the plausible method of the bat's echolocation was derived. The important evidence of the theory presented here is the fact that only the echolocating bats of the species *Vespertilionidae*, that emit the frequency-modulated pulses can detect the fish under the water surface. The bats of the another species *Rhinolophidae* emit the pulses with a constant amplitude and frequency, and they are not a fish-eating bat (3).

The method of the bat echolocation is explained here in terms of the interaction of the frequency-modulated acoustical pulse with the water layer between the swimbladder and the water surface. It was shown, that in the presence of the swimbladder the amplitude of the echo reflected by the water surface depends on the signal frequency. In the usual range $50-100 \text{ kHz}$ of the echolocation there are at least four regions of frequency for which the amplitude of the echo reflected back to the bat varies up to 25%. Thus, the bat can detect fish in water easily.

**REFERENCES**