Spectral Diffusion of Seismo-Acoustic Waves in Shallow Water

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Abstract: The propagation of seismo-acoustic waves in a strongly forward scattering medium is modeled as an energy diffusion process satisfying a convection-diffusion equation in slowness space. No energy propagates with a slowness greater than the Stoneley/Scholte wave slowness; therefore there is a zero flux boundary condition at the maximum slowness. At a slowness equal to 1/(lower half-space shear speed), energy is lost by radiation to shear waves, and for finite frequencies satisfies a mixed impedance type boundary condition in slowness space. The result is that energy diffuses from mode to mode until it is eventually radiated away as shear waves into the lower half-space. The diffusivity is a complicated functional of the auto- and cross-correlations of the horizontal material parameter gradient fluctuations. Numerical computations show that this spectral diffusivity can be a strong function of frequency and slowness with the result that energy in certain group slowness windows will propagate over greater distances than energy in other windows with greater diffusion. This leads to a signal coda that does not monotonically decay with time; an effect that has been observed in data.

INTRODUCTION

This paper addresses the modeling of an acoustic signal in a complicated shallow water environment. Specifically, we are interested in the case when the propagating acoustic energy is all forward scattered. We assume the environment can be modeled as a layered medium: fluid layers for the water column and elastic layers for the sediments and basement. The layers may vary in range, may have randomly rough interface boundaries and may contain random volume heterogeneities. The model is terminated at depth by a homogeneous half-space. The random properties are specified through a correlation function.

The seismo-acoustic field is represented locally as a superposition of modes. The range dependence and randomness scatter the propagating energy, coupling the local modes as the signal propagates. We model this scattering as modal diffusion in spectral space. Subject to a number of assumptions, the energy satisfies the convection-diffusion equation (1)

\[
\frac{\partial E}{\partial x} = \Gamma \frac{\partial E}{\partial p_x} + D \frac{\partial^2 E}{\partial p_x^2},
\]

where \(E\) is the elastic wave energy, \(x\) is the range coordinate, \(p_x\) is the horizontal slowness, and

\[
\Gamma = \rho^{-1} \frac{\partial}{\partial p_x} \left[ \frac{b(p_x,x)}{\rho(p_x,x)} \right]
\quad \text{and} \quad
D = \rho^{-2} b(p_x,x) \frac{\partial}{\partial p_x} \frac{b(p_x,x)}{\rho(p_x,x)}
\]

are the convection "velocity" and diffusivity, respectively. The function \(b(p_x,x)\) is the mode density in slowness space and \(b(p_x,x)\) is a function related to the autocorrelation of the mode coupling matrix and is discussed further below. The primary assumption involved is that forward scattering is dominant. Other assumptions and a detailed derivation of Eq. (1) are given in reference 1.

The boundary conditions for Eq. (1) are

\[
E(p_{\text{min},x}) + K(\omega) \frac{\partial E(p_{\text{min},x})}{\partial p_x} = 0 \quad \text{and} \quad \frac{\partial E(p_{\text{max},x})}{\partial p_x} = 0.
\]

The upper boundary condition is a zero flux condition. This arises because there are no propagating waves at slownesses greater than the slowness of the Stoneley wave propagating at the water-sediment interface. We are neglecting any contribution from the non-propagating modes.

The lower boundary condition is a mixed condition of impedance type containing the frequency dependent function \(K(\omega)\). Kohler and Papanicolaou(2) have a clear discussion of this boundary condition for their one-layer fluid problem. The most important thing about the function \(K(\omega)\) is that it vanishes as \(\omega \to \infty\). At this point the lower boundary condition becomes purely lossy. The lower edge of the slowness domain corresponds to the value \(p_x = 1/\text{shear speed}_{\text{half-space}}\), i.e., the shear wave slowness of the lower homogeneous half-space. Because we have assumed a medium in which the coupling is dominated by nearest neighbor interactions, the energy diffuses from higher values of slowness to lower slowness values until it radiates away into the half-space as a shear wave.

Construction of the function \(K(\omega)\) requires knowledge of the radiation modes for a layered fluid-elastic half-space. Maupin(3) has recently shown how to construct the improper eigenfunctions of the continuum part of the Green's function for the fluid-elastic problem. In deriving \(K(\omega)\), we have used Maupin's representation of...
the improper modes. Since our model is terminated by a homogeneous elastic half-space, we require the expression for the improper mode comprising trapped compressional waves in the upper layers and a traveling shear wave in the half-space. Because of the complexity of the expressions, we omit the details here.

SPECTRAL DIFFUSIVITY

As mentioned above the spectral diffusivity \( D \) is related to the autocorrelation of the mode coupling matrix, and is a complicated functional of the auto- and cross-correlations of the material property range gradients. Figure 1 shows the diffusivity for fluctuations in the elastic moduli at 15 Hz and 20 Hz for both a 1 meter and a 100 meter correlation length.

![Figure 1](image)

FIGURE 1. The spectral diffusivity for a realistic shallow water model comprising a water layer, low shear speed sediment layers and a higher shear speed underlying basement. The correlation function is exponential. Notice the strong frequency and slowness dependence. Comparing the curves for correlation length scales it is clear that rougher environments (\( l = 1\text{m} \)) are more efficient diffusers than smoother environments (\( l = 100\text{m} \)).

DISCUSSION AND CONCLUSIONS

Notice in Figure 1 that the spectral diffusivity is clearly frequency and slowness dependent. Although we do not show it, the diffusivity is also sensitive to the type of material perturbations. The most important feature is the deep notch for the 15 Hz figure. The diffusivity drops by two orders of magnitude for a slowness of 3.2 s/km. This indicates that energy propagating with this slowness is likely to remain coherent for greater distances than energy at other slownesses for this model.

The spectral diffusivity may be a strong function of slowness and frequency. Non-monotonically decaying codas are often observed in recordings of strongly scattered signals. Off path scattering has been offered as an explanation for this phenomenon (4). However, under the circumstances described by our model, energy propagating with certain group slownesses (velocities) may suffer somewhat less scattering and propagate more coherently over a greater distance than energy propagating with less favorable group slownesses. When one considers the way in which different modes sample a medium, it is clear that scatterers concentrated around a node of a particular mode will have much less influence on that particular mode. Relatively less scattering at certain group slownesses will produce a time domain signal that is not monotonically decreasing.

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REFERENCES