Aspects of cylindrical shell resonances in the Fourier diamond spaces.
Use of Surface Wave Analysis Methods (S.W.A.M.)
on experimental or numerical data.

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Abstract: The resonant aspect of a cylindrical shell and the propagation of a damped surface wave are studied in a monodimensional medium using the Fourier diamond spaces. The properties of the Impulsional Méthode d'Identification et d'Isollement des Résonances (M.I.I.R.) are demonstrated for the first time. A numerical simulation is proposed for a given cylindrical shell immersed in water. In a second part, the Ksi resonant wave number-frequency representation is experimentally illustrated.

MODELISATION OF THE SHELL SPACE-FREQUENCY REPRESENTATION

In this first part the propagation of a surface wave on an hollow cylindrical shell is modelised using the Fourier diamond spaces [1]. The incident plane wave generates each frequency of the surface wave for an incidence angle $\alpha(\omega)$ (Figure 1). Only the contribution of the propagating Surface Acoustic Wave (SAW) is considered: the SAW emits energy in the $\alpha$ direction during its propagation around the shell. The SAW is characterized by its complex wave number $K(\omega)=K'(\omega)+jK''(\omega)$. For small attenuation $K''$, the SAW can make more than one revolution and is then observed in multiple time positions for a given observation position $x$. The corresponding spectrum $S^-(x,\omega)$ is the sum of the spectrum of each echo. The phenomenon is the same for the SAW propagating in the other sense and characterized by their space frequency representation $S^+(x,\omega)$. The total spectrum observed for a given position $x$, is then:

$$S(x,\omega)=S^+(x,\omega)+S^-(x,\omega)$$

When the considered SAW is not too much attenuated,

$$S(x,\omega)=\frac{\cos(Kx)}{\sin(K\pi a)}$$

for $0 < x < \pi a$, and

$$S(x,\omega)=\frac{\cos(K(x-2\pi a))}{\sin(K\pi a)}$$

for $\pi a < x < 2\pi a$

The modulus of $S$ as a function of $x/(2\pi a)$ and $K1a=\omega a/c1$ is performed for A-wave (Figure 2: upper left corner). It can be shown that:

- $|S(x=\text{cte}, K1a)|$ is maximum when $K1a$ is such that $K'a$ is an integer (Figure 2: down left corner): the frequency resonances are observed.

- $|S(x, K1a=\text{cte})|$ has $n$ maxima (Figure 2: upper right corner): the real $n$ mode is identified.

In a simple way, the Impulsional M.I.I.R. has been demonstrated for the first time. For low frequencies, these method is well adapted. When frequency resonances are not
separated or when spatial interferences are not possible, this method is not adapted. A new method is proposed which allows the complete identification of the observed SAW.

EXPERIMENTAL SHELL WAVE NUMBER-FREQUENCY REPRESENTATION

The wave number-frequency \( \kappa(x,\omega) \) representation of the waves is the spatial Fourier transform of \( S(x,\omega) \). The experimental signals are collected around a cylindrical duralumin shell \((b/a=0.9)\). The experimental wave number-frequency \( \kappa(x,\omega) \) representation is performed (Figure 3). Four interesting new points can be demonstrated [2]. These points experimentally verified here are:

1. the positive and negative SAW are separated in the corresponding positive and negative \( K'a \) quadrants
2. when the SAW is not too much attenuated, the Regge [3] trajectories are defined: \(|Ks| \) is maximum for \( K'a \) integer (good correspondance with theoretical values \( 000 \)).
3. in this last case, a cut of \(|Ks| \) versus \( \omega \) is an \( \omega \)-Breit-Wigner function which exhibits the frequency resonances and the corresponding half-widths.
4. when the wave is highly attenuated, the \( \kappa(x,\omega) \) representation allows the identification of the wave space and time characteristics, resp. complex \( K \) and \( Q \).

In all the cases, S.W.A.M. can fully identify the wave characteristics.

CONCLUSION

Using the Fourier diamond spaces, the resonant aspect is modelised and experimentally verified. As new results, the Surface Wave Analysis Methods presented in a companion paper [4] can be used in order to identify the complete space and time wave characteristics. In addition, the wave sense of propagation is also identified in simple way.

REFERENCES