Boundary-Integral-Equation Methods for Accurate Calculation of Acoustic Fields

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Abstract: A boundary-integral-equation (BIE) technique has been used to calculate the acoustic fields in a Greenspan viscometer. The viscometer consists of a circular duct coupled at the ends to two concentric cylindrical chambers. The BIE code provides for the exact representation of axi-symmetric shapes whose cross-sections consist of mixtures of straight line and circular arc segments. Numerical solutions to the integral equation were determined by using cubic-spline approximations to the velocity potential Φ. This procedure enabled both Φ and the tangential acoustic velocity Φ’ to be determined with high precision. In calculations of the fields within ducts near the orifices, with modest (~200) numbers of boundary elements, approximate values of Φ’ were found which were smooth over 5 orders of magnitude. The technique was used to calculate fields near the duct ends and to evaluate inertial and resistive end effects, for both baffled and unbaffled duct ends. The effects of rounding sharp edges has been evaluated through the use of circular-arc boundary elements.

Acoustic fields in a cavity can be approximated by solutions of the Helmholtz equation subject to Neumann boundary conditions on the boundaries

$$\nabla^2 + k^2 \Phi(r) = 0, \ r \in C; \ \frac{\partial \Phi}{\partial n} = 0, \ r \in S;$$

(1)

where Φ is the velocity potential. For axi-symmetric geometries, this boundary-value problem can be expressed as a one-dimensional integral equation

$$-Ω(t')\Phi(t') = \int g_n(t',t)\Phi(t)h(t)r(t)\,dt.$$  (2)

Here $g_n(t',t)$ is a suitably defined kernel, $t$ is a parameter describing the boundary shape $r(t), z(t)$, $Ω(t)$ is the internal solid angle at point $t$, $ds = h(t)dt$ is an element of arc length in the cavity cross-section, and the velocity potential is now expressed as a function of $t$.

A simplified version of the geometry investigated is shown in Fig. 1. Approximate solutions to Eq (2) were found in the form of piecewise cubic polynomials whose first and second derivatives with respect to arc length $s$ were continuous nearly everywhere on the boundary. The exceptions were reentrant corners where the solutions are expected to have weakly singular tangential derivatives $Φ'$. An series expansion of $Φ$ near such corners was smoothly joined to the cubic on nearby boundary elements.
Solutions near the orifice of one viscometer are shown on the right of Fig 1. The solutions $\Phi(t)$ were calculated as described above, and Green's theorem was used to calculate internal values of $\Phi(r, z)$. The inertial and dissipative effects of the diverging flow in the orifice were parameterized by defining a duct end impedance and two related lengths

$$Z_{end} = \frac{\rho \omega}{\pi r_d} [i \delta_I + (i + 1) \delta_R].$$  \hfill (3)

The real part of this was obtained as the ratio

$$R_{end} = (P - P_0)/U_{end}^2, \quad P = \frac{\pi}{2} \mu \omega \delta_v \int (\Phi')^2 r ds.$$  \hfill (4)

Here $P$ is the total viscous dissipation in the cavity, $P_0$ is the dissipation in a length $L_d$ of an infinitely long duct, and $U_{end}$ is the volume flow through the duct orifice. The imaginary part of Eq (3) was determined from the numerically-determined eigenvalue and the acoustic model of the Greenspan viscometer. (1) The inertial end correction $\delta_I$ and the resistive end correction $\delta_R$ were obtained for many Greenspan viscometer shapes as functions of the dimensional parameters defined in Fig 1. (2) Recent calculations focused on the dependence of $\delta_I$ and $\delta_R$ on the wall thickness. Some typical results are shown in Fig 2. The inertial end correction decreases with relative duct thickness, qualitatively consistent with the limit $\delta_I \approx 0.61 r_d$ for an infinitesimally-thin duct radiating into infinite space. (3) The resistive end correction increases with decreasing relative duct thickness. Other numerical tests with $r_d'/r_d = 1.5$ show that when the insertion length $L_i$ is varied, $\delta_I$ has a broad minimum near the center of the chamber, and $\delta_R$ has a broad plateau near the center of the chamber with a minimum value at $L_i = 0$, and increasing values as $L_i \to L_c$. Recent experimental determinations (4) of the end corrections for ducts with $(r_d' - r_d)/r_d$ near 1.3 and 1.5 are within a few percent of the calculated values.

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