Linear Resonant Duct Thermoacoustic Refrigerator having Regenerator Stacks

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Abstracts: Measurements of acoustically generated temperature difference across the TAC (thermoacoustic couple) in a speaker driven closed tube are made as a function of the sound frequency and the position of the TAC. During the measurements the electric power input supplied to the speaker was kept constant. The position dependency of the temperature difference across the TAC follows predictions based on a theory by Wheatley et al. [J. Acoust. Soc. Am. 74, 153-170(1983)]. The peak acoustic pressure amplitude, determined by matching these theoretical predictions to each of the experimental data sets, varies with sound frequency in a manner of resonance character. This resonance is attributed to the source resonance. The source characteristics and the position derivative of the temperature difference across the TAC lead to a formula for the determination of the optimum positions of the stacks.

THEORETICAL BACKGROUND

The temperature gradient-supporting stack(regenerator) is the key element of thermoacoustic engines. The effects, on engine performance, of the stack position relative to the acoustic source, however, received little attention in the literatures. Therefore, we have undertaken a study on stack position dependency of engine performance in a simple thermoacoustic engine consisting of short stack referred to as a Thermo-Acoustic Couple(TAC)[1] in a loud speaker driven closed tube. The utility of this choice is that the theory can be reduced to its simplest form for analysis of the results. The first measurements of the thermoacoustic effect using a TAC were performed by Wheatley et al. [1]. They derived an expression for a steady-state temperature difference developed across a TAC in an acoustic standing wave. Atchley et al. [2] made measurement on acoustically generated temperature difference for a wide range of drive ratio(the ratio of the peak acoustic pressure amplitude to the mean pressure of the gas) from 0.1% to 2.0% and compared the results with predictions based on the theory by Wheatley et al.

\[ \Delta T = - \frac{A(1+\sqrt{\alpha})}{(1+\alpha)} \sin 2\pi x \left[ 1 + \frac{A(1-\cos 2\pi x)}{\sqrt{1 - A(1-A)/(1+\alpha)}} \right] \]  

where \( A = \rho_\infty c_s \left( \frac{k_{p\alpha} + k_{p\omega}}{4\rho_\infty c_s} \right) \), \( x \) is the TAC position, \( \Delta x \) is the TAC length. Thermal and viscous penetration depths are defined as \( \delta_s = \sqrt{2k_{p\alpha} \rho_\infty c_s} \) and \( \delta_\omega = \sqrt{2\mu/\rho_\infty} \), respectively. The Prandtl number is defined as \( \alpha = c_\mu K = (\delta_s/\delta_\omega)^2 \), \( d_p \) and \( d_s \) are the thickness of stack plate and the thickness of gas layer respectively. Atchley et al. found three distinctive phenomena: first, agreement between measurements and theory is, in general, best in the vicinity of acoustic particle velocity nodes at all drive ratios investigated; second, for drive ratios less than approximately 0.4% measurements agree well with the theory; third, for higher drive ratios the measured temperature difference is much less than that of the predicted result. The first and second are the cases for which the denominator in Eq. (1) is nearly unity, and the temperature difference across the TAC is a nearly sinusoidal function of its position, while the third case, the increase in the denominator with TAC position is significant and the temperature difference changes from the nearly perfect sinusoid to something closer to a sawtooth shape with the maximum(minimum) shifted to be near the pressure antinodes. These suggest that the temperature distribution may not be linear across the stack, but may be a skew-S shape with constant temperature plateaus at both ends of the TAC. Therefore we propose the TAC length \( \Delta x \) in Eq. (1) to be replaced by the temperature gradient-supporting column of the stack \( \Delta x \), which is only a fractional length of the TAC, \( a \Delta x \). If Eq. (1), with \( \alpha \) in it, was verified, its alternative maxima and minima correspond to the most efficient positions at which the stack pumps heat forward and backward, respectively. The position derivative of Eq. (1) leads to the forward pumping stack positions \( x_f = (n+1)\lambda/2 - x_0 \), and to the
backward pumping stack positions \( x_b = n \lambda/2 + x_0 \), where \( n \) equal 0, 1, 2, \( \cdots \), \( \lambda \) is the sound wavelength and \( x_0 \) is given by

\[
x_0 = \frac{\lambda}{4\pi} \cos^{-1} \left[ \frac{\left( A/T\omega b\right)(1 - \sigma^2)/(\gamma - 1)(1 - \sigma^2)}{1 + \left( A/T\omega b\right)(1 - \sigma^2)/(\gamma - 1)(1 - \sigma^2)} \right].
\]

(2)

**EXPERIMENTAL INVESTIGATION**

In order to test the significance of \( a \) in Eq. (1), we made measurements of acoustically generated temperature difference across the TAC in a loud speaker driven closed tube as a function of the sound frequency and the position of the TAC. The TAC is made of a squarely porous ceramic, machined 29 mm i.d. and 37 mm long. A thermopile, consisting of ten of type K chromel-alumel thermocouple junctions connected in series, is epoxied on front and back faces of the TAC. The TAC is mounted on the end of a hollow 1/4 in o.d. stainless steel tube TAC probe with surface graduations of 12 mm intervals used for stack position measurements. The horn driver generates standing acoustic wave in 68 cm long PVC tube rigidly capped and drilled for the passage of the TAC probe. During the measurements the electric input power supplied to the horn driver was kept constant.

![Figure 1. \( \Delta T \) vs 2x/\( \lambda \). Symbols, measurements; Solid curves, predictions of Eq. (1) with \( a = 1/3 \).](image1)

![Figure 2. The resonance curve of the source. \( \omega_0 \), experimentally deduced values; Solid curve, resonance equation.](image2)

The position dependency of the temperature difference across the TAC is shown in Fig. 1 for various acoustic frequencies. The solid curves are predictions of the modified Eq. (1) with \( a = 1/3 \) which fit the experimental data sets very well. In matching Eq. (1) to each of the experimental data sets in Fig. 1, the amplitude of the peak acoustic pressure \( \psi \) or the value of \( A \) in Eq. (1) was deduced. Fig. 2 shows the frequency variation of \( A \). The solid curve is the predictions of a standard form of the resonance equation

\[
A = A_{max}/[1 + Q^2(f/f_0 - f_0/f)^2]
\]

with \( f_0 = 219 \) Hz and \( Q = 3.44 \). This resonance phenomenon is tributable to the source resonance. This source characteristic, frequency variation of \( A \), can be used in Eq. (2) for the determination of the optimum stack positions \( x_t \) and \( x_s \) in a double stack thermoacoustic refrigerator.

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**REFERENCES**