Studies in bowing point friction in bowed strings

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Abstract: A method of determining the frictional force of bow on string is described. It uses the force on the terminations of the string, assumptions about the propagation of waves on the string, and measured reflection functions of the waves at the terminations. Comparison with simulations and some experimental results are shown, as well as discussion of errors from assumptions not entirely fulfilled.

The frictional force between bow and string in bowed string motion is the only quantity of major importance not directly measurable. At least one measurement of the frictional properties of rosin has been reported (1), but it was not made on the bowed string dynamical system. Reference (1) shows that the frictional force diminishes monotonically with increasing relative velocity. In the absence of the friction force measurements in a realistic dynamical context, the standard assumption has been that the result of (1) applies for bowed string motion. This assumption has been used in the many computer simulations (2,3) of the bowed string. One measurable consequence of the assumption that the frictional force is a single valued function of the relative bow-string velocity is the existence of a threshold for flattening (4,5). That has been found compatible with experimental results (5,6). The only study of rosin-mediated frictional forces on a dynamical system was by Smith (7), on a single degree of freedom system. The present work extends (7) to the bowed string.

The frictional force as a function of time can be obtained from appropriate combination of the forces on the string's terminations. The method is related to tomography in that it infers or derives an internal property from an external measurement. As applied here, there are a number of restrictions and approximations that limit the accuracy of the result. These will be carefully discussed in this presentation.

Let $u_0$ be the outgoing velocity wave from the bow and $u_i$ the incident wave from a termination, both at time $t$. The incident wave and $u_i$ was created at the termination at $t-\Delta$; $u_0$ is incident on the termination at $t+\Delta$, where $\Delta$ is the propagation time from bowing point to termination. We assume a non-dispersive string. The force of the bow on the string is:

$$F_b(t) = Z(u_0 - u_i)$$

(1)

The velocity waves incident on and generated by the bow can be written in terms of the forces at the termination at the delayed and advanced times $t-\Delta$ and $t+\Delta$. At time $t+\Delta$ the force the termination exerts on the string is:

$$F_i(t+\Delta) = Z(r \ast u_0 - u_0) = Z(r(t) - \delta(t)) \ast u_0 = Z\rho(t) \ast u_0$$

(2)

where $u_0$ is the same quantity as in eq. (1) because of the non-dispersion assumption, the symbol "$\ast$" denotes convolution, $r(t)$ is the reflection function for a wave at the termination, and $\rho = r - \delta$, where $\delta$ is the Dirac delta function.

Let $v_0$ be the outgoing wave generated at the bow at $t-2\Delta$. The force on the string at the termination at $t-\Delta$ is:

$$F_i(t-\Delta) = Z(u_0 - v_0) = Z(r \ast v_0 - v_0) = Z\rho \ast v_0$$

(3)

Both equations (2) and (3) must be solved for $u_i$ and $u_0$ in terms of the termination forces by deconvolving. If we denote that process by $\rho^{-1}$ and use $u_i = r \ast v_0$, then we can write the bow force as

$$F_b = \rho^{-1} \ast F_i(t+\Delta) - r \ast \rho^{-1} \ast F_i(t-\Delta)$$

(4)

Of course at the bow there are waves generated in both directions, towards the bridge and towards the nut. Consequently the total force on the bow is the sum of two terms of the form of eq. (4), with separate delays appropriate to bridge and nut, and the reflection function $r(t)$ also appropriate to those terminations. In addition, the velocity at the bowing point of the string in terms of the advanced and delayed termination forces can be found with similar manipulations; again, $v(t)/Z$ is the sum of two expressions with the appropriate delays and reflection functions. The terms in the velocity expressions are those in eq. (4) except that the second term on the rhs of eq. (4) is added to the first, rather than subtracted.

The equations for $F(t)$ or $v(t)/Z$ are the solutions of inverse problems. If the reflection functions are delta functions, the solutions are algebraic, and there are no further restrictions on achieving a solution. Lacking that unachievable experimental condition, a solution may be obtained by Fourier transformation, utilizing the convolution theorem. It is, however, impractical to carry out this procedure for a data run consisting of a single bow stroke, thus going from zero frictional force and string velocity initially to that same condition at the end. The alternative is to choose a
portion of the run that is as nearly periodic as possible and perform the discrete Fourier transform (DFT) on just that
data set. The data is obtained by sampling, and the consequent discreteness and the periodicity assumption lead to
the following procedure for reconstructing the bow force and bowing point velocity from the termination forces.
a) The selected excerpt of the data several periods long (typically 4 to 8) is resampled so that there are an integral
number of samples per period. The measured reflection functions must also be resampled at the new rate.
b) A criterion is used to select the propagation delays from bow to bridge and nut. These delays will in general not
be an integral number of samples, so the resampled data must be again resampled to obtain the value of the
termination forces at non-integral sample times. The criterion that seems most successful is to minimize the
spread of velocities during the sticking portion of the cycle. The resamentals are done using interpolation in
\textit{Mathematica}.

This procedure has been tested with a variety of simulated data, for which the resultant bow forces and bowing
point velocities are of course known from the simulation. Knowing the reconstructed velocity and force as a
function of time allows the determination of reconstructed data pairs \((v,F)\) that should lie on the friction force vs.
string velocity curves used in the simulations. The accuracy of the fit is very sensitive to the delays used. The most
reliable criterion for delay selection, based on simulated data experiments, is to minimize velocity spread during
sticking, and to insist that the average string velocity be zero. The most serious errors that occur after the delays
have been selected are from the bandwidth limitations imposed by the sampling rate. At the standard CD rate, events
that occur at time scales shorter than about 50 \(\mu\)s are not revealed. The rapid transition from sticking to slipping is
thus not resolved at that sampling rate.

For real data, the most damaging errors in reconstructing the friction force are caused by the periodicity
requirement. Real data is not periodic. Consequently the frictional force reconstructed from the DFT procedure
with \(n\) periods of data is the force needed to make the string repeat those \(n\) periods indefinitely. That obviously can
differ from the actual force exerted on the string in the experiment.

Another assumption is that the wave shape does not change during propagation to and from the terminations.
While the propagation changes can validly be subsumed in \(r(t)\) for the purpose of simulations, they must in principle
be included both before and after convolution with \(r(t)\) to do the reconstruction properly. That step has not yet been
implemented. To minimize dispersion from string stiffness, the data has been taken entirely on E strings.

In the presentation we will compare reconstructed friction vs. velocity curves for simulated data with the input
force vs. velocity of the simulations. Reconstructions using real data from violin E strings bowed at a “point” (50
\(\mu\)m to 100 \(\mu\)m of length on the string) with a rosined glass bow will be presented as well. Since this system is a
useful model system for studying friction in multi-mode dynamics, there will be discussion of the implications for
friction research, as well as suggestions for modification of the standard friction function paradigm in modeling.
Finally, since the dynamics of real bows enter in the forces exerted on the string – the glass rod is presumed to have
no significant dynamics – we will show data obtained with a real bow in which the short term bow dynamics at
capture and release seem to play an identifiable role.

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**References**