Physical Parameter Uncertainty Effect on Frequency Dependent Acoustic Response in Ocean Sediment

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Abstract: The physical parameter uncertainty effect on the Biot-Stoll poroelastic wave response is investigated. Analytical formulae for statistical moments are established. Results related the use of the Biot-Stoll theory in light of input parameter uncertainty and statistical variability are presented.

INTRODUCTION

The problem of acoustic waves propagating in the ocean over a porous seabed is of interest in many ocean acoustic applications. Considering the sediment as fluid filled porous media, the Biot-Stoll poroelasticity theory has gained popularity. Its application requires the knowledge of a minimum 13 physical parameters. While the demand of information is more than the traditional theory, a practical procedure for estimating the parameters using a minimum geological and geotechnical data was recently presented. It is known that whenever a parameter is estimated, uncertainty exists. This paper quantitatively examines the effect of input parameter uncertainty on the output uncertainty in terms of statistical moments.

PARAMETER UNCERTAINTY ANALYSIS

The dynamic acoustic response of a porous media can be characterized by three complex velocities, $V_{P1}$, $V_{P2}$, $V_{S}$. The real parts of these three velocities are the propagation velocity, and the imaginary parts correspond to attenuation. The dynamic behavior of saturated sediment is influenced by the seepage characteristics of oscillatory flow in pores, controlled by frequency $f$, porosity $\phi$ and permeability $k$. An investigation of the parameter uncertainty effect on the acoustic response and the reflection and transmission coefficients is conducted here, the results are provided for the solid elastic as well as hydraulic properties. The effect of pore size distribution is also examined.

Shear Modulus Effect on the Acoustic Response The statistical structure of a data set taken from the AGS (Atlantic Generating Station) site reveals that the shear modulus is lognormally distributed. Based on the statistical information, we establish an analytical technique to analyze the parameter effect on the acoustic response. We define $s = \log G$. Let the mean and the standard deviation of $s$ be $\mu_s$ and $\sigma_s$, respectively, we obtain the mean and the variance of the acoustic response as functions of $\mu_s$ and $\sigma_s$ using the analytical statistical moment formulae developed earlier. As a demonstration, the mean value of $s$ is chosen for a 'hard sediment' case, $\bar{s} = 7.4186$. We examine the shear modulus effect on the acoustic response at standard deviation $\sigma_s$ as 3.5% of $\bar{s}$, which is corresponding to a standard deviation of $G$ over 45%. Detailed results are shown in Mu, et al. Generally we find that, in the whole range of the concerned frequencies, $V_{P1}$ and $V_{P2}$ are not sensitive to the shear modulus uncertainty. The shear wave velocity and its attenuation are very sensitive to the shear modulus uncertainty.

Porosity Effect on Propagation Velocities Through several empirical relations, it has been demonstrated that parameters such as shear and bulk modulus, permeability, etc. are related to porosity. For example, the shear modulus can be given by the relation:

$$
G = \alpha_1 \left(\frac{1 - \phi}{\phi}\right)^{a_2} \left(g(1 + 2K_0)(\rho_s - \rho_f) \int_0^\phi [1 - \phi(z)] dz\right)^{1/2}
$$

In the formula, $g$ is the gravitational acceleration, $\rho_s$ is the grain bulk density, $\rho_w$ is the pore water density, and $K_0$ is the coefficient of earth pressure at rest, $\alpha_1$ and $\alpha_2$ are experimental constants dependent of the sediment type, and $\alpha_1 = 2.44 \times 10^5$ and $\alpha_2 = 1.628$ are suggested. The permeability can be calculated using the Kozeny-Carman equation. The frame bulk modulus can be estimated from:

$$
K = \alpha_1 \cdot \exp(-\alpha_2 \phi)
$$

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in which the empirical constants $a_1$ and $a_2$ are obtained from the experimental data. Hamilton\textsuperscript{7} suggested $a_1 = 5.121 \times 10^{10}$, $a_2 = 9.795$ for marine sands and $a_1 = 5.443 \times 10^{10}$, $a_2 = 9.788$ for silty clay. The above relations allow us to infer a number of parameters from one parameter that is obtained from a certain field engineering measurement, such as the vane shear test or standard penetration test\textsuperscript{8}.

The porosity uncertainty effect on the acoustic response is examined under conditions that the Biot moduli, permeability, shear modulus and the frame bulk modulus are all related to the porosity. This study can be used as the reliability analysis of the procedure provided by Badiey et al.\textsuperscript{2} Assuming that porosity is normally distributed with a mean $\bar{\phi}$ and a variance $\sigma_\phi^2$, the statistical mean of the response velocities and their variances are derived analytically according to the statistical moments theory. Considering the hard sediment\textsuperscript{4} with $\bar{\phi} = 0.47$ and $\sigma_\phi$ at 10\% of $\bar{\phi}$, the results are presented in Figure 1. The dashed lines are ± standard deviation envelopes. We also notice the frequency dependent features in the plot.

**Fig. 1** Illustration of porosity uncertainty effect.

**Fig. 2** Pore size distribution effect on $R$.

**Pore Size Distribution Effect on Acoustic Response and Reflection Coefficient** The effect of pore size distribution is an important issue in Biot theory. It affects the equivalent permeability and the apparent viscosity of the oscillatory flow. We investigate its effect on the general acoustic response and on the reflection coefficient. To study the effect of the pore size distribution, we consider a log-normal distribution\textsuperscript{8}. For the complex velocities, we find that the statistical moments of the pore size have minor effect at lower frequencies. Hence the pore size distribution effect is not responsible for the higher than predicted attenuation observed at lower frequencies\textsuperscript{5}. The squirt flow model on the other hand, may account for this effect. To study the effect of pore size distribution on the reflection coefficient ($R$), we examine the plane wave reflection at the water-sediment interface at different frequencies and all incident angles. Figure 2 shows such results. We find that the reflection coefficient is highly affected by the pore size distribution after the critical incident angle.

**REFERENCES**