A Comparison of Hankel Transform Algorithms’ Performance for Use in Shallow Water Applications

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Abstract: Complex acoustic pressure data as a function of range may be decomposed into its constituent horizontal wavenumber components by taking the Hankel transform. This paper investigates the accuracy of a number of discrete Hankel transform algorithms applied to computer simulated pressure data in a shallow water channel for three typical seabed bottom types. The results are compared against the exact solution and the most accurate algorithm identified for shallow water applications. The differing behaviour of these algorithms is illustrated by their use in the Hankel transform method proposed by Frisk G.V. et al. [J. Acoust. Soc. Am. 68(2), 602-612 (1980)] for the determination of seabottom plane wave reflection coefficients.

TESTING THE ACCURACY OF THE HANKEL TRANSFORM ROUTINES

In a shallow water channel the complex acoustic pressure field \( p(r,z) \) may be decomposed into the horizontal wavenumber components of the depth dependent Green function \( g(k_r,z) \) via the Hankel transform (HT) of the pressure measured over range. The HT relationship between \( p(r,z) \) and \( g(k_r,z) \) is given by the transform pair:

\[
g(k_r,z) = \int_0^\infty p(r,z) J_0(k_r r) r \, dr, \quad p(r,z) = \int_0^\infty g(k_r,z) J_0(k_r r) k_r \, dk_r \quad (1a,b)
\]

Although the integral has an upper limit of infinity, in practice the pressure field will become negligible at some finite range. Of the routines proposed in the literature for taking HTs, only some of those transform uniformly sampled data, as might be measured from, for example, a towed array. Eight routines that fit this requirement are considered here. The ability of each of the eight routines to perform an accurate HT was tested using synthetic acoustic pressure data. The synthetic data was calculated from the analytically known Green function for an iso-speed waveguide, for three different bottom types (clay, sand, and limestone), by using high-precision adaptive wavenumber integration of equation 1b. This test data was then Hankel transformed (equation 1a) using the routines, and the results obtained compared to the exact Green function. Figure 1, which shows the results for the sand-bottom case, exhibits two points of interest. First, as expected, the error in the retrieved Green function becomes independent of the range of data transformed beyond a certain range. Second, the routine with consistently the least error is that derived by Oppenheim et al. (1). The same features were seen in the results from the other bottom types which suggests that this routine is the most accurate for use in shallow water acoustics applications.

APPLICATION TO REFLECTION COEFFICIENT DETERMINATION

The need for an accurate Hankel transform routine is illustrated in an application to determine the reflection coefficient of the sea-bed. For the prediction of acoustic pressure in shallow water it is necessary to know the plane wave reflection coefficient as a function of angle or horizontal wavenumber. A method of calculating this reflection coefficient from the depth dependent Green function has been proposed by Frisk G.V. et al. (2). Using this method, estimates of the reflection coefficients corresponding to the three test bottom types were calculated using the Green functions retrieved by the different HT routines. It is clear from figure 2 that the choice of HT routine can, in this application, have a significant effect on the estimate of the reflection coefficient.
FIGURE 1. The weighted, normalised, rms error in the retrieved Green function as a function of range of data transformed for the different HT routines for the Green function for the sand-bottomed case. Inset: the theoretical and the most accurate retrieved Green functions for the maximum range. This shows that, given sufficient range, there is one HT routine which is the best for shallow water applications.

FIGURE 2. The exact reflection coefficient for the sandy bottom case and the best and worst estimates of it calculated from the retrieved Green functions, showing that the choice of Hankel transform routine can be vital in some applications.

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REFERENCES