Sequential versus iterative methods for recovering acoustical impedance profiles of inhomogeneous media

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Abstract: A sequential method which lies on a new algorithm for computing the reflection response of a truly layered medium for arbitrary input and an optimization method for identification of the same medium are reported. The results of both methods are compared for some synthetic echoes. The optimization technique has the drawback of being more time-consuming. On the other hand, the sequential method is very fast. Both have, at least for the numerical experiments conducted up to now, a very close behaviour when identification is performed under noisy data.

INTRODUCTION

This work presents a comparison between two time-domain sequential inverse methods for evaluation of either smooth or non-smooth impedance profiles of transversely infinite media excited by plane waves at normal incidence. In opposition to frequency-domain methods [1], this approach is suitable for identification using arbitrary input signal and its echo. The first method in the comparison [2] is a sequential one, based on the model of Tenenbaum and Zindeluk [3], where the profile is recovered layer by layer. The second one is based on a formulation of the identification problem as an optimization procedure and makes use of minimization methods to recover the original profile. A small numerical experiment is carried out to evaluate their performances and their sensitivities when identification is performed using noisy data.

SEQUENTIAL METHOD

The problem to be solved is, knowing the sampled components of both signals, the incident, \( F_j \), and the reflected one, \( G_j \), from an inhomogeneous obstacle, to determine its acoustic impedance profile, \( Z_i \). It can be shown that the reflection coefficients, \( R_i \), can be recovered using the following expression [2],

\[
R_i = \frac{Y_i - Y'_i}{V_i^0 \prod_{k=1}^{i-1} (1 - R_{i-k})},
\]

where the polynomials \( Y_i \), \( Y'_i \) and \( V_i^p \) come from the general solution for the direct problem, see [2,3]. The times required for the computation of both cases (smooth and nonsmooth profiles), are of the same order of magnitude than those of the direct problem.

ITERATIVE METHOD

The iterative method is based on the idea of minimizing the distance between the measured or simulated reflected signal and a synthetic one, calculated for an arbitrary profile. One can define a cost function, which can be, using an Euclidean norm,

\[
S = \sum_{i=0}^{M} \left( \| F_i, R_i \| - \| G_i \|^2 \right)
\]

where \( M \) is the number of components used in the calculation. One have then a function \( S : \mathbb{R}^M \rightarrow \mathbb{R} \) which represents the distance between a synthetic (\( \mathcal{L}(F_i, R_i) \)) and the measured response. The vector \( \mathbf{R} = (R_0, R_1, \ldots, R_N) \) which yields a null or minimum distance is the set of reflection coefficients which correspond
to the solution, being necessary that
\[
\mathbf{R} = \{R_1, R_2, \ldots, R_n\} \in \Omega,
\]
where \( \Omega \) is the feasible region, where all entries of \( \mathbf{R} \) lie in the interval \([-1,1]\).

In order to avoid the difficulty imposed by the excessive time consumption of the second derivatives calculations needed when using, for instance, Newton's method, one can use methods known as secant methods [4,5,6], that is, those which generate approximations of the hessian matrix generated as the algorithm progresses, as the BFGS algorithm [5,6]. One of the advantages of this method lies on the lesser evaluations required of the cost function. For each entry of the hessian matrix, four evaluations are necessary; if the problem involves \( N \) variables, \( 2N^2 \) evaluations will be necessary, while with a BFGS approximation one needs only \( 4N + 2 \). The updating rule of course needs an initial guess, which is, usually, the identity matrix.

**NUMERICAL EXAMPLES**

The following figures show a stratified profile identified using both methods described above. Fig. 1 shows the identification performed using noiseless incident and reflected signals. Fig. 2 shows the same profile, but now identified with white noise added to both signals.

**CONCLUSIONS**

The iterative method took 116 s. to solve the problems with noiseless signals while the sequential one took only 1 s. The results of identification with noisy data suggests that both algorithms have very close degree of instability and, thus, at this moment, one is not able to establish the more advantageous for such a situation. The time consumption, however, points to the sequential method as the first choice for this kind of problem.

**REFERENCES**


