Waveguide Characterization Estimation in Shallow Water by a Vertical Array

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Abstract: Waveguide characterization estimation in shallow water using a complex mode approximation of acoustic field is presented. The experimental results for \( f = 500 \text{Hz} \) acoustic signal source which located at \( z_1 = 11 \text{m} \), \( r_1 = 21 \text{km} \) show that the extracted accuracy of \( k_m \) is between \( 10^{-4} \) and \( 10^{-5} \), the related error of \( \beta_m \) is between 9% and 26%.

INTRODUCTION

In recent years, the inverse problem of extracting the environmental information from the measured field data has been rapidly developed, and many methods using the horizontal line array or synthetical apertures have been proposed. In the present paper, we introduce a method to estimate the waveguide characteristic parameters \((k_m, \beta_m)\) using a vertical array, specially when the structure of bottom medium is less known or unknown.

THEORETICAL ANALYSIS

It is well known that in shallow water the acoustic pressure emitted by a harmonic point source of unit source strength at long range is the sum of excited modes (ignoring time factor \( e^{-i\omega t} \))

\[
P(r, z) = i\pi \sum_m U_m(z)U_m(z_0)H_0^{(1)}(\mu_m r)
\]

where \( \mu_m \) is complex horizontal wavenumber of the \( m \)th mode, imaginary part of which is defined as the mode attenuation coefficient \( \beta_m \). \( U_m(z) \) is the normalized eigenfunction. In case of homogeneous water layer, when \( \beta_m \) is known, \( U_m(z) \) can be approximately given by

\[
U_m(z) = \frac{2}{H_e} \sin(\sqrt{\left(\frac{\omega}{c_0} + i\alpha\right)^2 - \mu_m^2 z})
\]

where \( H_e \) is complex effective depth of homogeneous waveguide, which is equal to the sum of the bottom complex effective depth \( \Delta H \) and the water depth \( H \). For a long range acoustic field corresponding to the low grazing angle, \( H_e \) is approximately a constant. For an inhomogeneous water layer, \( U_m(z) \) can be obtained from WKB approximation. Hence, equation (1) comprises the complex mode approximation of acoustic field.

In addition, \( \mu_m \) can be expressed with \( H_e \) as

\[
\mu_m^2 = \left(\frac{\omega}{c_0} + i\alpha\right)^2 - \frac{m^2\pi^2}{H_e^2}
\]

Eq.(1) shows that the effect of the environmental parameters on acoustic field is mainly determined by the waveguide complex effective depth \( H_e \) and the mode number \( m \). It means that the acoustic field at low grazing angle can be calculated, as long as the waveguide complex effective depth \( H_e \) and the mode number \( m \) are known, and the fine structure of bottom medium, such as the number and thickness of layers, the velocity of compressive and shear wave and attenuation, is unknown.

Supposing the complex pressures \( P_n \) received by a vertical array which contains \( N \) hydrophones have been measured, the effective mode number \( M_e \) and the waveguide complex effective depth \( H_e \) can be solved by the L-S method.

\[
\epsilon = \sum_{n=1}^{N} |P_n - P_n'|^2
\]
where \( P_n \) is the complex pressure calculated by Eq. (1). Then we can extract waveguide parameters \((k_m, \beta_m)\) by using Eq. (3) and deduce the approximate equation of the reflection coefficient \( V(k) \) [2], which corresponds to low-grazing angle \( \theta \left( 0 \leq \theta \leq \theta_c, \theta_c \) is the critical angle \( ) \):

\[
V(k) = \frac{\gamma \Delta H - i}{\gamma \Delta H + i}
\]

where \( \gamma \) is vertical wavenumber in water layer.

**EXPERIMENTAL RESEARCH**

The environmental condition and the signal source position are shown from Fig.(1), which are taken from literature[3]. Substituting complex values of acoustic pressures in 13 channels into Eq. (4), the waveguide effective mode number \( M_e = 4 \) and complex effective depth \( H_e = (29.36, 0.002) \) are obtained. Extracted waveguide parameters \( k_m \) and \( \beta_m \) are listed in Table 1. For the signal of \( f = 500 \text{Hz} \), the waveguide cutoff mode number is \( M_e = 8 \). The first four waveguide parameters \( k_m \) and \( \beta_m \) obtained by MOATL program[4] are listed in Table 1 as reference values. It can be seen that the deviation of \( k_m \) is about \( 10^{-4} - 10^{-5} \), and the related error of \( \beta_m \) is between 9 and 26%.

**TABLE 1. Extracted waveguide parameters and errors \( (f=500 \text{Hz}) \)**

<table>
<thead>
<tr>
<th>Mode NO.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{m0} (1/\text{m}) ) ( \text{[MOATL]} )</td>
<td>2.123167</td>
<td>2.115167</td>
<td>2.101666</td>
<td>2.082383</td>
</tr>
<tr>
<td>( k_m (1/\text{m}) )</td>
<td>2.123120</td>
<td>2.115051</td>
<td>2.101438</td>
<td>2.082281</td>
</tr>
<tr>
<td>( \beta_{m0} \text{[neper/m]} ) ( \text{[MOATL]} )</td>
<td>-4.7E-5</td>
<td>-1.16E-4</td>
<td>-2.28E-4</td>
<td>-1.02E-4</td>
</tr>
<tr>
<td>( \beta_m \text{[neper/m]} )</td>
<td>3.5380E-6</td>
<td>5.5461E-6</td>
<td>8.3869E-6</td>
<td>11.702E-6</td>
</tr>
<tr>
<td>( \beta_{m0} - \beta_m \text{[rad/m]} )</td>
<td>3.2048E-6</td>
<td>4.3234E-6</td>
<td>6.2071E-6</td>
<td>8.8861E-6</td>
</tr>
<tr>
<td>( \beta_{m0} \text{[rad/m]} )</td>
<td>-0.0942</td>
<td>-0.2024</td>
<td>-0.2599</td>
<td>-0.2406</td>
</tr>
</tbody>
</table>

Fig.(2) shows the reflection coefficient \( V(k) \) by using Eq. (5). The solid curve stands for reference values, the dashed curve is approximate values.

**FIGURE 1. Ocean environment**

**FIGURE 2. The bottom reflection coefficient \( f = 500 \text{Hz} \)**

**REFERENCES**