A Discussion on Non-Radiating Sources

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Abstract: The structure of sources giving origin only to a local non-propagating field is discussed. The analysis shows that
these sources appear as the difference of two source distributions possessing the same field outside a given region and can be
basically of two types. It is pointed out that their importance lies in the fact that their identification in the mathematical
description permits advantageous simplifications in the source function, providing a better understanding of sound generation
mechanisms. The equation for the velocity field due to a dipole distribution is used to illustrate the discussion.

INTRODUCTION

The fact that a given sound field can be generated by more than one source distribution [1] is quite well known but is
frequently given an over exaggerated importance or misinterpreted. As pointed out by Ffowcs Williams [1], for the
inhomogeneous d'Alembert equation,

$$
\nabla^2 \varphi(x, t) = \left( c_0^2 \frac{\partial}{\partial t} \right) \varphi(x, t) = Q(x, t)
$$

where the source term Q is zero outside a closed volume V, adding \( \int V \nabla \varphi \) to both sides of (1), where \( \nabla \) is any
suitable function satisfying \( \nabla(0) = 0 \), generates, outside \( V \), no difference in the value of the resulting field, \( \varphi + \nabla \varphi \),
with respect to the original one, although the source has changed. Doak [2], on the other hand, pointed out that the
causal source for a given field is unique. The part of \( Q \) that produces a null field outside \( V \) and which might be
thought responsible for any ambiguity, \( Q_{R} \), he called a reactive source distribution, or a source of silence, in
opposition to the active part, the source of sound. The present paper discusses the structure of these sources of
silence and their role in the study of sources.

DISCUSSION

The existence of the so-called ambiguity reflects a fundamental property of waves equations. The left side of a wave
equation can always be represented as the difference of two terms (or groups of terms), which can be seen as two
source distributions that produce the same field everywhere, except inside the source region [3]. In acoustics, where a
wave equation is obtained by combining the isentropic version of the continuity equation with the momentum
equation, these terms can be regarded as describing the generation of sound by a local isotropic expansion due to the
addition of mass or heat and due to the application of stresses. Outside the source region, the stresses are generated
by propagation, matching exactly (in the linear approximation) those produced by the local changes in volume, so that
the two fields are identical. Inside \( V \), their difference is non-zero, there existing an excess of stresses or of volume
changes, which gives origin to true sources. It is evident that, by examining the wave field, one cannot tell which of
the two processes originated the field.

In an homogeneous medium at rest, this “ambiguity question” refers basically to the impossibility of distinguishing
between the field of a volume displacement point monopole and that of an isotropic volume acceleration point
quadrupole (i.e., between the fields of source distributions of the form \( c_0^2 \frac{\partial}{\partial t} \varphi(x, t) \) and \( \nabla^2 \varphi \)). They differ only by a
local component, introduced by the \( \nabla^2 \) operator in the quadrupole source (note that, in three-dimensional space,
\( \nabla^2 (F \varphi) = - 4\pi \delta(x) \)). In a more general case described by a wave equation, a corresponding situation involves a
more elaborate combination of sources originating from the continuity and from the momentum equation, needed to
properly describe, in a non homogenous or non-uniformly moving medium, the sound generation processes referred
to above.

1929
The existence of a “source of silence” in a given problem requires that part of the source function can be written as the difference of two source distributions producing, outside \( V \), the same field. The case of a surface source distribution enclosing an inner source distribution was discussed by Doak [2]. This type of reactive source has important applications in active control but is not expected to be found naturally, i.e. unless intentionally designed, in the composition of source functions. The Ffowcs Williams type of “source of silence”, on the other hand, exists when part of the source term reproduces the structure of the wave operator, being written, for equation (1), as \( Q_r = \nabla^2 \psi \) for some \( \psi(x, t) \) that is zero (or negligible) outside \( V \), and, for more general wave equations, represented by \( \Delta \phi = Q \), as \( Q_r = \Delta \psi \). Source functions with this characteristic do appear in the algebra of source terms and their importance lies in the fact that the identification of the reactive component and its and removal from \( Q \) optimizes the description of sound generation processes, permitting a better insight into the physics involved.

Goldstein [4], for instance, showed that, by ingenious manipulation, part of the source terms of Lilley’s equation, which describes sound generation and propagation in a parallel sheared flow, can be rewritten in the form of the wave operator acting on a quadratic quantity. This shows that these terms are not relevant to the sound generation process, their transference to the right hand side simplifying the source description.

Another interesting example is given by the velocity field of a volume acceleration dipole distribution in an homogeneous medium at rest, a point that is often mishandled. It is simple to show that the wave equation for velocity \( v \), when source terms per unit mass \( q \) and \( f \) are considered in the continuity and momentum equations, respectively, can be written as

\[
\Box^2 v(x, t) = -\nabla(q - \nabla \times F),
\]

\( F(x, t) = \int f(x, r) \, dr \) (2, 3)

The \( \nabla \times \nabla \times F \) term is frequently regarded as a problem, having been discarded by Jessel [5] and, more recently, by Lasota [6], on the assumption that it is reasonable to suppose that \( \nabla \times f = 0 \) (what guarantees that the \( v \) field is irrotational everywhere). Since this is not at all true for a general dipole distribution, both authors found, for a point dipole, a velocity field that is rotational everywhere, what is obviously incorrect.

Since \( \nabla \times \nabla \times F = \nabla \nabla \cdot F - \nabla^2 F \), equation (2) can be rewritten as

\[
\Box^2 v = -\nabla(q - \nabla \cdot F) + \Box^2 F \quad \text{or} \quad \Box^2(v - F) = -\nabla(q - \nabla \cdot F) \quad (4 \text{a, } 4 \text{b})
\]

showing that the terms related to \( f \) in (2) hide a “source of silence” whose identification advantageously simplifies the source function. The velocity field described by (4.a) is irrotational everywhere, except where \( \nabla \times F \) is non-zero, what can happen only in the source region.

The form (4.b) evidences that the propagating part of the velocity field induced by a dipole distribution \( f \) is dependent only on the irrotational part of \( f \). It may also appear to suggest that the full \( f \) will be responsible for a local, non propagating field but, since the local component in the field of \( \nabla \nabla \cdot F \) cancels exactly that due to the irrotational part of \( \Box^2 F \), it follows that only the solenoidal part of \( f \) contributes to the local field. Thus, for an homogeneous medium at rest, although vorticity does not, in the linear approximation, generate propagating effects, it generates a reactive source for velocity, since \( \nabla \times v = \nabla \times F \). The analysis of Lefebvre [7], based on the equations for the solenoidal and irrotational parts of \( v \), leads to the same conclusion. The present approach offers the advantage that it does not require calculating the irrotational component of \( f \) for obtaining the radiation field. It also shows that, in the wave equation for velocity, in spite of what equation (2) may suggest, the source terms \( q \) and \( f \) have a very similar role to the one they have in the equation for pressure.

REFERENCES