Simulation of ultrasound propagation in a thermally turbulent fluid using Gaussian beam summation and Fourier modes superposition techniques

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Abstract: A numerical model has been developed to simulate the propagation of ultrasonic waves within thermal turbulence. It uses the random Fourier modes method, for generating thermal fields and Gaussian beam summation, to calculate the acoustic field propagated in the fluid.

INTRODUCTION

For control and inspection of Nuclear Power Plants, the French Atomic Energy Commission has developed ultrasonic telemetry and imaging devices as well as models for simulating them. A numerical model was developed to simulate the propagation of ultrasonic waves within a two dimensional homogenous isotropic thermal turbulence. This is applied in this paper in order to determine the fluctuations of phase and amplitude from signals sent between a given point transmitter and a receiver. The statistical results are obtained by calculating the acoustic fields for a large number of realizations of turbulent temperature fields, generated by the summation of a finite number of random Fourier modes [1]. To model the acoustic propagation we used the Gaussian beam summation method which enables easy calculation on caustics and in shadow zones of geometrical acoustics.

FOURIER MODES SUPERPOSITION TECHNIQUE

The temperature fluctuation $T'(x)$ at the point with coordinates $x$ is written in the form of a sum of a finite number of Fourier modes:

$$T'(x) = \sum_{n=1}^{N} \tilde{T}_n \cos (k_n \cdot x + \phi_n)$$

where the directions of the wave vectors $k_n$ and the phases $\phi_n$ are random variables uniformly distributed over the interval $[0, 2\pi]$ in order to respect the field's isotropy and homogeneity. The amplitudes $\tilde{T}_n$ are linked to the energy spectrum of the turbulence. We assume a Gaussian spatial correlation function, $R(r) = <\tilde{T}(r) \cdot \tilde{T}^*(r)> \exp(-r^2/L^2)$.

GAUSSIAN BEAM SUMMATION

For each realization of the temperature field, the calculation of the 2-D acoustic field is performed in the following way.

First step: ray tracing.
Second step: calculation of the contribution $u_\nu$ of each ray to the pressure at receiver $M$. By solving a local high frequency approximation of the parabolic equation, Cerveny [2] obtains:

$$u_\nu = \Psi \left( \frac{c(s)}{q(s)} \right)^{1/2} \exp \left[ -i \phi \left( \tau(s) + \frac{1}{2} \frac{p(s)}{q(s)} \right) \frac{MH_\nu^2}{c(s)} \right]$$

(2)
where $\alpha$ is the angle of the ray direction, $\Psi$ a constant, $c$ the speed of sound, $\omega$ the angular frequency, $H_a$ the projection of $M$ on the ray, $s$ the arc length along the ray. The complex numbers $p$ and $q$ are the solutions of the dynamic ray tracing system:

$$\frac{\partial p}{\partial s} = cp \quad \frac{\partial q}{\partial s} = -\frac{1}{c^2} c^2 q \frac{\partial^2 c}{\partial n^2}$$

where $\partial / \partial n$ denotes a partial derivative along the direction normal to the ray.

Third step: Summation of the contributions of all the rays in order to calculate the total pressure. The formalism of the Fourier transform permits at this stage to synthesize time signals (pulses). Our algorithm allows the detection of the existence of eigen rays and to calculate their travel time.

**PRELIMINARY RESULTS**

We present results of digital simulations for the following conditions: propagation with an average water temperature of 30°C (wave celerity of 1509 m/s), standard deviation of the temperature fluctuations of 3.6°C (standard deviation of the acoustical index is 5.5 $10^{-5}$), parameter $L$ of the Gaussian spatial correlation function of 3 cm, acoustical frequency $f = 600$ kHz (wave length is 2.5 mm). The study variable is the $X/L$ ratio, where $X$ is the propagation distance. Results are described for the variance $<\Delta t^2>$ in travel times and for the scintillation index $\sigma_i^2$.

In the region of small fluctuations, corresponding to the lowest values of $X/L$, the results obtained for $<\Delta t^2>$ closely agree with Chernov's analytical solution and those for $\sigma_i^2$ closely agree with Rytov's analytical solution, as shown by figure 1.

In the case of strong fluctuations, the method predicts a non linear change in $<\Delta t^2>$, but from $X/L$ values that exceed those predicted by Karweit [3]. Regarding $\sigma_i^2$, the method does in fact predict saturation but at a level that is lower than expected.

The simulations in the time domain for various realizations show very clearly signal amplifications or extinctions as well as signal splitting associated to the existence of multiple acoustic micropaths between the emitter and the receiver.

In a second paper presented to this Congress, we have applied the model in order to show the influence of turbulence on array defocusing and on the use of time reversal techniques to correct this defocusing[4].

**REFERENCES**