Abstract An elastic continuous structure is coupled with a number of oscillators, distributed in some frequency range. Even in the limit of vanishing damping it gives the finite value of spatial decrement (Landau damping). The cases of real energy damping and storing the energy in the oscillators system are analysed. It is shown that if the external harmonic excitation has a decrement it is possible to organize an spatial enlightenment.

A system consisted from master structure - an oscillator, or a waveguide coupled with a number of oscillators, distributed in some frequency range, is considered. It is possible to describe the damping in such system as «Landau damping» [1,2]. Its principal feature, described for the first time by Landau [1] - the finite decrement even when the damping in the oscillators tends to zero. The reality of the damping is still being discussed. Let us analyze this problem following Landau method, using the quasiharmonical excitation, growing exponentially from zero for \( t > \infty \). If the distribution of the oscillators in the eigenfrequency range is \( f(\omega_n) \), then the governing equations are:

\[
\begin{align*}
\rho y_n - \alpha y_n &= F(x,t),
\rho y_n &= -\int f(\omega_n) g(\eta) [y(x,t) - y_n(x,t)] d\omega_n,

m_n \xi_n + m_n \beta \xi_n + g_n [\xi_n - y(x,t)] &= 0.
\end{align*}
\]

Here \( \rho, T \) - density and tension of the string, \( g_n, m_n \) - rigidity of the spring and mass of the \( n \)-oscillator. \( \beta_n \) - the dissipation coefficients, accordingly, in string and in oscillators, connected with it, \( y(x,t), \xi_n(x,t) \) - dynamical displacements of the string and the oscillators, \( \omega_n^2 = g/m_n \) - the square of the oscillators eigenfrequency, \( c^2 = T/\rho \) - the square of wave velocity in a homogeneous string. The first integral of the system (1) is:

\[
\frac{\partial}{\partial t} \left[ \rho y_n^2 + \frac{T y_n^2}{2} + \int \left[ \frac{m_n \xi_n^2}{2} + g_n (y - \xi_n)^2 \right] f(\eta) d\eta \right] - \frac{\partial}{\partial x} ( Ty_y + ) + \beta y_n^2 + \int g_n [\xi_n - y(x,t)] f(\eta) d\eta = 0.
\]

The expression (2) is the energy conservation law, which is fulfilled in every point of the string \( x \) in every moment \( t \), the expression in figured brackets is the energy density \( E \), the expression under gradient - the density of the energy flux along the string, the last two terms are describing the nonreversible dissipation of the energy \( E \) in the same point \( (x,t) \). The system is determined on the half-infinite interval

\[ 0 < x < \infty, \quad -\infty < t < \infty. \]

An fixed source acts at the point \( x = 0 \), describing the switching on with the small increment \( \delta \) at \( t = -\infty \) according Landau [1]:

\[ y(0,t) = y_0 \exp(-j\omega t + j\delta t), \quad \delta > 0. \]

The solution corresponding to the boundary condition (3) for arbitrary \( t > 0 \) is:
\[ \zeta_n(x,t) = \frac{\omega_n^2}{\omega_n^2 - (\omega + j\delta)^2 - j\beta_n \omega} - y(x,t), \]

\[ y(x,t) = y_0 \exp(-j\omega t + \delta \tau + jkx), \]

here \( k \) is being found from the expression:

\[ k^2 = \frac{(\omega + j\delta)^2 + j\beta \omega}{c^2} + \frac{\int f(n)(m_n/\rho c^2)}{(\omega_n^2 - (\omega + j\delta)^2 - j\beta_n \omega) \partial \omega_n}. \]

The imaginary part of \( k \) is:

\[ \text{Im}k = \frac{\eta}{2k_0} \int f(n)(m_n/\rho c^2)(\omega_n^2/2k_0) \left[ \frac{2\omega_0^2}{(\omega_n^2 - \omega_0^2)^2 + \omega_0^2 \eta_n} \right] \partial \omega_n. \]

The real damping \( \beta \) and increment \( \delta \) give the contribution to the coefficient of spatial damping of the field \( \eta \) of the same sign. The expression in square brackets under the integral if \( \eta_n \to 0 \) tends to \((\pi \omega/2)\delta(\omega_n - \omega)\). Finally, it gives:

\[ \text{Im}k = f(n) \frac{\eta \omega^2}{4\rho c^2 k_0} \partial \omega_n. \]

\( \eta \) is determined by relation \( \omega_n = \omega \) and \( \text{Im}k \) becomes independent from \( \eta, \eta_n \). In spite of the same sign of the contribution to \( \eta \) of \( \delta \) and \( \beta \), their role is essential different. The vibrations are growing in every point \( x > 0 \) with the increment \( \delta \). If to change the sign of \( \delta \), when for example \( \delta = 2\beta \), the spatial decrement \( \text{Im}k \) vanishes. This is the effect of enlightenment. The changing of the sign of \( \beta, \beta_n \) is impossible, this sign is given in (1) by the thermodynamical condition of dissipation. Naturally, if \( \beta = 0 \) the process is reversible: if starting from \( t_0 > 0 \) to change the sign of \( \tau \) for all \( t = t_0 + \tau \) in the boundary condition (3):

\[ y(0,t_0 + \tau) = y_0 \exp(\xi - j\omega \delta)(t_0 + \tau). \]

then the phase point in the phase space of the whole system will move strongly back along the phase trajectory. The energy flows from the fuzzy system of oscillators back to the string. Following the Poincare theorem of return [3], the system will go back periodically if \( \eta = 0 \), when the boundary condition (3) becomes zero, but the time of return is exponentially growing with the number of degrees of freedom and is infinite for continuous system. The reversible processes described above are impossible in strong sense if the real damping exists. The deviation from reversibility is finite and is governing by the damping factor (6) even if \( \beta, \beta_n \) tends to zero. It can be shown that the same damping factor acts when the system is exited by delta-impuls instead of (3).

References.