Theoretical analysis of reactive silencers with two propagating modes

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Abstract: A general modelling of sound ducts with reactive (or dissipative) treatments considers one main guide and a secondary one (the treatment). For a duct lined with a periodical treatment, an expression for solutions of dispersion equation (infinite length) and insertion loss (finite length) is available. Continuous limit and non-local reaction effects are discussed.

INTRODUCTION

The sound propagation is studied in a waveguide containing two different propagation media separated by a rigid perforated screen parallel to the propagation axis. An example of such a system is a waveguide lined on its walls with a rigid framed porous material behind a perforated screen. All the transverse dimensions are much smaller than the wavelength. The discrete model is presented in a previous paper [1]. Firstly, for periodic liners, a thorough study of the dispersion equation shows how the behaviour of non-locally reacting liners is directly related to the number of eigenmodes modes propagating in the system. Secondly we show for a perforated tube muffler, how non-local reaction and interferences between this two propagating eigenmodes, can achieve resonant maxima for the insertion loss coefficient of a finite lengthed system.

BIMODAL WAVEGUIDES

Let consider an one-dimensional and periodic infinite system with no flow. This system is reciprocal and can be chosen symmetric. It is intuitive that the transfer matrix of any portion of the two coupled waveguide is of order four, relating the two acoustic pressure and velocities in each waveguide. At low frequencies the pressure discontinuity on each side of the opened areas of the perforated screen coupling the two propagation media can be ignored [2]. Thus the effect of one perforation is described by a single lumped admittance $Y$, which behaviour is like $\sigma/(j\omega L)$, where $j^2 = -1$, $\omega$ is the pulsation, $\sigma$ is the open area ratio of the perforated screen and $L$ is the added mass associated to one perforation of the screen. The two propagation constants $\Gamma$ and $\Gamma'$ of the eigenmodes are solutions of the following dispersion equation [2]:

$$ (ch\Gamma - A_1 - \gamma_2 Y B_1)(ch\Gamma - A_2 - \gamma_1 Y B_2) = Y^2 \gamma_1 \gamma_2 B_1 B_2, \tag{1} $$

where $S_1$ and $S_2$ are the cross section areas of each guide, and $\gamma_i = \frac{2\pi i}{S_1+S_2}$ with $i = 1$ or 2. As an example, if waveguide 1 is rectilinear and filled with air of density $\rho$ and sound velocity $c$, then we have simply $A_1 = \cos(k_1 l)$, $B_1 = j\rho c \sin(k_1 l)$, with $k_1 = \frac{2\pi}{l}$, and $l$ is the periodicity of the lattice. Of course $A_i$ and $B_i$ ($i=1$ or 2) should be different to take into account the presence of some discontinuity and/or porous material in the waveguides 1 and 2. At any frequency the dispersion equation (1) has two solutions $ch\Gamma$ and $ch\Gamma'$. We point out that as far as we are concerned with reactive phenomena, the study is limited the lossless case. Thus all coefficients of eq. (1) are real, the solutions $ch\Gamma$ and $ch\Gamma'$ are real, and frequencies at which $-1 < ch\Gamma < 1$ corresponds to the propagating eigenmode $\Gamma$. It can be shown that at very low frequency only one eigenmode is propagating : the plane mode with equal acoustic pressure and velocities in both waveguides, and the second eigenmode is evanescent (called the "flute mode" in reference [2]). This latter becomes propagating at a cutoff frequency which is increasing with open area ratio of the screen $\sigma$. This cutoff is identical to the one given in reference [3] p 356, when both propagation media are identical (classical perforated tube muffler). Thus the low frequency domain is separated by this cutoff: below the cutoff the number of propagating eigenmodes is 1, i.e. NPM=1, and above the cutoff, NPM=2.

DISCUSSION

Let first examine the case of small coupling between the waveguides. For sufficiently high frequency values or small open area ratio $\sigma$, the terms containing $Y$ in eq.(1) are small and the two eigenmodes $ch\Gamma \simeq A_1$ and $ch\Gamma' \simeq A_2$ are the planar modes of each waveguide. If $\sigma$ is small, the cutoff frequency is shifted towards zero (NPM=2 even at very low frequency), but the waves tend to propagate independently in each waveguide. Let now study the case of a strong coupling. If the perforations of the screen are so wide that the screen is
vanishing (continuous limit), the admittance $Y$ is no more valid but a qualitative reasoning shows that we can assume $A_1/(\gamma_2 Y B_1)$ and $A_2/(\gamma_1 Y B_2)$ to be small. The evanescent eigenmode $c h \Gamma' \simeq Y (\gamma_1 B_2 + \gamma_2 B_1)$ can be ignored. The propagating eigenmode $c h \Gamma \simeq (\gamma_1 B_2 A_1 + \gamma_2 B_1 A_2)/(\gamma_1 B_2 + \gamma_2 B_1)$ is a planar mode depending on characteristics of both propagation media (and not on $Y$). As far as one eigenmode can be ignored (NPRI=1), a wall admittance depending on both media $Y_w = 2(A_1 - A_2)/(B_1 + B_2(S_1/S_2))$ is found, and the system can be considered as an wave guide lined with a locally reacting acoustic treatment. In case we have $\xi_1 \ll \xi_2$ and $\xi_2 \ll \xi_1$, the wall admittance per unit of length reduces to $Y_w = j \omega S_z/(\rho_2 c_2^2)$ which represents the compressibility of a volume and does not depend on medium 1. This latter can be seen as a waveguide lined with a porous material (e.g. [4]), which behave as a locally reacting treatment. The elastic-like behaviour of the liner decreases the cutoff frequencies of the transverse mode of guide 1 [5] and do not achieve sound attenuation at low frequency because viscous losses are negligible in the liner. Finally, it is interesting to examine a finite lengthed system featuring a frequency band with two propagating eigenmode (NPRI=2). We add diaphragms (in guide 2) to a configuration given by Sullivan [6]. The geometry of the silencer and the corresponding insertion loss are given on figure 1. On the left of the dashed lined (i.e. below 2000 Hz), only one eigenmode is propagating in the system. The insertion loss is of an expansion chamber. Maxima are non resonant and their amplitude is given by the ratio $r_{\text{out}}/r_{\text{in}}$; their periodicity corresponds to the length of the total system. On the right of the dashed line (i.e. above 2000 Hz) the non local reaction induces that two eigenmodes are propagating in the muffler (NPRI=2). It can be shown that $\Gamma$ and $\Gamma'$ are close one to another and interferences of the eigenmodes produce resonant maxima for the insertion loss (their amplitude depends on viscous losses), and their periodicity is also related to the total length of the finite system.

CONCLUSION

Systems which exhibits frequency band with two propagating eigenmodes corresponds to non locally reacting liners (bimodal waveguides). The approach discussing the number of eigenmodes propagating in the system is shown to be very usefull to predict the characteristics of a wide class of bimodal waveguides, of infinite or finite length.

REFERENCES