Giant Response in Dynamics of Small Bubbles

Iskander Sh. Akhatov, Claus-Dieter Ohl *, Robert Mettin *, Ulrich Parlitz * and Werner Lauterborn *

Ufa Branch of Russian Academy of Sciences, K. Marx Str. 6, Ufa, 450000, RUSSIA

*Drittes Physikalisches Institut, Universität Göttingen, Bürgerstraße 42-44, D-37073, Göttingen, GERMANY

Abstract: For small bubbles in a strong sound field a strong response with a thresholdlike increase in oscillation amplitude exists. This phenomenon has a great influence on many properties of cavitation bubbles. The following aspects are considered: rectified diffusion, positional stability of the bubble under SBSI conditions, primary and secondary Bjerknes forces.

For single bubble oscillations under medium and large pressure amplitudes a complicated scenario of bifurcations and coexisting (possibly chaotic) attractors exists (1). However, for very small bubbles in very strong sound fields the dynamics becomes more regular and a strong response with a thresholdlike increase in oscillation amplitude occurs (2). The physical reason for this "giant resonance" is the fact that for very small bubbles the surface tension pressure $P_\sigma = 2\sigma/R_0$ is very high and the bubbles behave like flexible solid particles even for large driving pressures. For this case, the normalized bubble radius $R(t)/R_0$ of a typical air bubble oscillation in water is shown in Fig. 1 (middle). Figure 1 (top) shows the driving pressure of the external sound field $p_s(t) = -P_\sigma \sin(\omega t)$. When we increase the size of the bubble $R_0$ it starts to oscillate differently (Fig. 1 (bottom)). During the expansion period the influence of the surface tension decreases rapidly, and therefore, the amplitude of the expansion grows enormously leading to a strong collapse. The transition point may be called nonstatic Blake threshold. When the equilibrium radius of the bubble is increased further, the influence of the surface tension pressure $P_\sigma$ becomes smaller and a nonmonotonous response curve for the normalized maximum radius $R_m/R_0$ occurs (Fig. 2 (top)).

![Figure 1](image1.png)  
**Figure 1.** The influence of surface tension ($P_\sigma = 2\sigma/R_0$) on bubble oscillations for pressure $P_\sigma = 1.5$bar. (top) driving pressure. (middle) $R_0 = 1\mu$m. (bottom) $R_0 = 1.5\mu$m.

![Figure 2](image2.png)  
**Figure 2.** (top) Response curves for different pressure amplitudes $P_\sigma = 1.1 - 1.5$bar showing the giant resonance. (bottom) Gas concentration in liquid near the bubble wall ($c$).
The rectified diffusion growth rate of a bubble is closely related to the response curve (3). Figure 2 (bottom) shows the averaged gas concentration (\(\langle \frac{c}{T} \rangle \)) near the bubble wall vs. the equilibrium radius \(R_0\). For small and medium values of \(P_a\) the concentration decreases monotonically as a function of the equilibrium radius. For sufficiently large amplitudes \(P_a\), however, the corresponding concentration curves possess a global minimum for small bubble radii. This nonmonotonic dependence of \(\langle \frac{c}{T} \rangle \) on \(R_0\) is a result of the strong response shown in Fig. 2 (top). The growth rate depends on the difference between the concentration of gas in the liquid \(c_\infty\) and the averaged gas concentration near the bubble wall \(\langle \frac{c}{T} \rangle \). If \(c_\infty\) is large (upper dashed line in Fig. 2 (bottom)), a single equilibrium radius \(R_{th}\) with \(\langle \frac{c}{T} \rangle = c_\infty\) exists that is unstable. This case is denoted in Fig. 2 (bottom) by the open circle at the point of intersection of the upper dashed line with the concentration curve for \(P_a = 1.2\) bar. Bubbles with radius \(R_0 < R_{th}\) dissolve due to diffusion flux from the bubble into the liquid. Bubbles with \(R_0 > R_{th}\) grow permanently due to rectified diffusion until they become very large (and may disintegrate). If \(c_\infty\) is small (lower dashed line in Fig. 2 (bottom)), two equilibrium radii with \(\langle \frac{c}{T} \rangle = c_\infty\) exist. The left one, \(R_{th}\), denoted by the open circle in Fig. 2 (bottom) is unstable. The equilibrium point plotted as a filled circle at the right-hand side in Fig. 2 (bottom) is stable and provides a stable radius \(R_s\) for single bubbles oscillating in the acoustic field. Bubbles with radius \(R_{th} < R_0 < R_s\) grow until they reach the stable radius \(R_s\). If the bubble radius is larger than \(R_s\) the bubble shrinks until \(R_0 = R_s\). This result has provided an explanation for the existence of small stably oscillating bubbles that have been observed in experiments on sonoluminescence.

The giant response phenomenon has an influence on the primary Bjerknes forces that are responsible for the positional stability (trapping) of a small oscillating bubble in the vicinity of the acoustic pressure antinode (4). For small \(R_0\) (before the resonancelike response occurs) the Bjerknes force is small and attractive. When increasing the equilibrium radius the response of the bubble increases rapidly leading to a strong amplification of the Bjerknes force. For medium \(P_a\) the Bjerknes force depends on \(R_0\) monotonically, but for \(P_a > 1.65\) bar it starts to depend on \(R_0\) nonmonotonically and for a certain value of \(R_0 = R_0^{crit}(P_a)\) the Bjerknes force changes sign and becomes repulsive. This means that for very strong amplitudes of the acoustic field only very small bubbles \((R_0 < R_0^{crit}(P_a))\) are trapped in the pressure antinode. The larger bubbles will be repelled because their position in the center of the flask becomes unstable.

The strong radial oscillations affect the mutual interaction of two bubbles in an acoustic field, the secondary Bjerknes force (5). The resonancelike response of bubbles leads to secondary Bjerknes forces that are stronger by factor of \(10^3 \sim 10^6\) than it may be expected from linear approximations. Additionally, the signs of the forces may change near the giant response region.

These results are important for understanding the trapping mechanisms in single bubble sonoluminescence and collective bubble phenomena such as streamer formation and multibubble sonoluminescence.

REFERENCES