Numerical Simulation with Experimental Validation for Nonlinear Standing Wave Phenomena

Zhichi Zhu, Anqi Zhou, Dongtao Huang, Xiaodong Bian* and Ke Liu**

* Dept. of Engr. Mechanics, Tsinghua Univ. and ** Institute of Acoustics, Academia Sinica, Beijing, P.R. China

Abstract: A numerical simulation for multiplying growth and saturation of higher harmonics in a nonlinear standing wave has been carried out. The entire development process of the nonlinear standing wave is predicted and some new interesting results are found. The numerical predictions are in good agreement with experimental data.

INTRODUCTION

Strong nonlinear standing wave phenomena are constantly encountered in the experimental studies of acoustics, such as multiplying growth of higher harmonics, saturation of harmonics, bifurcation, and chaos. However, the experimental study of strong nonlinear standing waves is highly restricted by the source, due to the difficulties associated in building up a source with a very strong sound power output. In this paper, we adopted a numerical approach to the phenomena of multiplying growth of higher harmonics and saturation of harmonics, and carried out a comparison between numerical result and measured data. The main goal of the paper is to illustrate the applicability and accuracy of the numerical method as well as to explore some phenomena which are hard to be identified even by modern acoustic measurements.

NUMERICAL METHOD

In order to compare with the experimental results presented by Ref. 1, the computation conditions are selected to be the same as that in Ref. 1. A constant acoustic pressure source $p' = p_0 \sin(\omega t)$ is imposed at the left end ($x=0$), where the velocity $u$ is assumed to be zero. Here $\omega$ is selected as the fourth resonance frequency of the tube. The right end ($x=L$) is strictly solid wall. The governing equations is 1-D matrix form Euler Equs. as follows:

$$\frac{\partial V}{\partial t} + \frac{\partial F}{\partial x} = 0$$

$$V = \begin{pmatrix} \rho \\ \rho u \end{pmatrix}, \quad F = \begin{pmatrix} \rho u \\ \rho u u + p \end{pmatrix}$$

where $\rho, p$ are density and pressure respectively, and they abide by isentropic relationship. MacCormack fourth order difference scheme is adopted (see Ref. 2). Once the end condition is satisfied, the numerical results of the sound field in the standing wave tube are obtained. The nonlinear standing wave phenomena are then investigated by using FFT technique for the acoustic pressure, whose strength is used to expressing as a sound pressure level, i.e. SPL or $L_p$.

NUMERICAL RESULTS

Fig. 1 shows the variation of relative sound pressure at $x=L$ in a resonance period for different SPL of excited source, which is oscillated in sinus at left end. The computational resonance frequency is 292.6 Hz, so the corresponding period is 0.003418 s. It is shown that the responding sound waves at right end are very different from sinusoid, when the SPL of the source exceeds 150 dB. They are combined by the basic frequency of 292.6 Hz and its higher harmonics shown in Fig. 2. The waves become distorted ones more and more, and the number of higher harmonics increases dramatically. It is noted from Fig. 2 that there exists a remarkable component of sound pressure at zero frequency in nonlinear acoustic waves. This is a new finding due to numerical simulation, which should be examined by experiments later.

**FIGURE 1.** Sound pressure wave at $x=L$.

**FIGURE 2.** Higher harmonics at $x=L$.
Fig. 3 shows the variation of SPL of harmonics with SPL of the excited source. It can be redrawn as an increment $\Delta$ of SPL of harmonics with SPL of the source shown in Fig. 4. It is seen clearly that there do exist phenomena of multiplying growth of higher harmonics and saturation of harmonics in the nonlinear cases.

**FIGURE 3.** Variation of harmonics with SPL of source  
**FIGURE 4.** Increment of harmonics with SPL of source

**EXPERIMENTAL VALIDATION**

The experimental results shown in Fig. 5 and 6 are adopted from Ref. 1. All of the experimental conditions are the same as the computational ones, but the frequency in experiment is 194 Hz, while 192.6 Hz, which is strictly the 4th order resonance frequency of the standing tube, is used to compute. The abscissa of Fig. 5 and 6 is driving voltage, whose maximum in experiments is 43 V, responding the SPL of 167 dB. It is concluded from the Fig. 3-6 that as a whole the agreement between the numerical results and the experimental data is very well.

**FIGURE 5.** Variation of harmonics with driving voltage  
**FIGURE 6.** Increment of harmonics with driving voltage

**CONCLUSIONS**

1. The multiplying growth and saturation of higher harmonics in nonlinear standing wave tube have been numerically simulated. With increasing the sound pressure level of the excited source from 125 dB to 191 dB, the entire development process of the nonlinear standing wave are clearly illustrated.
2. A detailed comparison between numerical simulation and previous experimental results shows that the numerical method and the relevant experiments are all successful.
3. Some new interesting results are obtained from the simulation. For example, the sound pressure level at zero frequency enlarges quickly with increasing the intensity of the excited source.

**ACKNOWLEDGMENT**

This study was financially sponsored by National Natural Science Foundation of China.

**REFERENCES**