Diffraction of a sound wave on open end of a waveguide with impedance walls and impedance flanges.

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Abstract: Diffraction of a plane sound wave on an open end of a waveguide with impedance walls, connected with impedance flanges is considered. A plane wave is incident upon the waveguide from open space. As a result of diffraction, a part of sound energy is scattered in the half-space, the other part of energy penetrates into the waveguide. Results of the solution of this problem can be used for calculation of the amplitude of a sound wave, scattered by a hole, in particular, in backward direction. Besides, one can calculate the sound energy, penetrating into the waveguide. Absorption of the energy in the waveguide can be taken into account by introducing impedances with nonzero real parts. The distortion of the directivity pattern of a system of receivers located in the waveguide can also be obtained. The solution of the problem has been carried out by the integral equation method using Green's function of the half-space with an impedance boundary. The equation was solved by expanding the unknown function in the eigenfunctions of the waveguide. It reduces to an infinite set of linear algebraic equations. Results of computation of bistatic diagrams of scattering on the hole and diagrams of backward scattering are presented.

Diffraction of a plane sound wave on an open end of a waveguide with impedance walls, connected with impedance flanges is considered (Fig. 1). The waveguide is connected with flanges also having a normal impedance, which can differ from the impedances of the walls of the waveguide. The waveguide can be either half-infinite or finite. In the latter case it can be considered as a deepening in a baffle. As a result of diffraction, a part of the sound energy is scattered in the half-space, while another part penetrates into the waveguide.

FIGURE 1. Diffraction of a sound wave on open end of a waveguide with impedance walls and impedance flanges.

a- half-infinite waveguide, b- finite waveguide (a deepening in a baffle)

The solution of such a problem is necessary in various fields of acoustics, in particular, to investigate backscattering of sound by an aperture, to calculate the sound pressure for radiation of a sound from an open waveguide, to investigate directivity properties of acoustical antennas situated in the waveguides and in the enclosures having sound absorption walls. In these cases one can calculate the distortions of directivity patterns of the arrays at scattering of the sound by walls and corners.

The classical solution of the diffraction problem for the sound and electromagnetic waves on an open end of a waveguide having Dirichlet or Neumann boundary conditions without flanges can be obtained by the Wiener-Hopf method is a well-known. The problems of this kind has been the subject of attention in extensive literature. The problem of diffraction of sound on an open end of waveguide having impedances walls was solved in Ref.1. However, for the waveguide with flanges this method is inapplicable. In Ref.2 a problem for the waveguide and flanges with hard surfaces has been studied. However the problem considered in the present paper is much more complicated. Scattering of a sound on real bodies is related to absorption of the energy. This process can be described by representations of a body surface as a boundary having complex impedance with positive real part.

One can describe the sound field in the lower half-space using the Helmholtz integral equation, Refs.(3,4)
\[ p(r_0, r_1) = -ik\rho c Q G(r_0, r_1) + \int_0^{2\pi} p(r) \frac{\partial G(r, r_1)}{\partial n} - \frac{\partial p(r)}{\partial n} G(r, r_1) \, dr \]  

Here \( G \) is the Green's function of the Helmholtz differential equation, \( r_0 \) and \( r_1 \) are the vectors defining location of the source and the receiver. For reducing this equation to the equation in an finite range it is useful to choose Green's function satisfying the impedance boundary condition on the baffle, i.e.

\[ G(r_0, r_1) = \frac{i}{4\pi} \int_0^{2\pi} \exp(iku(x_1 - x_0)) \left[ \exp(ik\gamma(y_0 - y_1)) + A_p(\gamma) \exp(-ik\gamma(y_0 + y_1)) \right] \, dr \]  

where \( \gamma = \sqrt{1 - u^2} \), \( A_p(\gamma) = (w_p\gamma - 1)/(w_p\gamma + 1) \). Here and below \( w \) with the corresponding indices denote the normalised impedances. The sound field in the waveguide is described as an expansions in the eigenfunctions \( \psi_n(x) \) satisfying the appropriate boundary conditions on the walls of the waveguide

\[ p_2 = \sum_{n=1}^{\infty} A_n \psi_n(x) \left[ \exp(ik\gamma_n) + qA_n(\gamma_n) \exp(-ik\gamma_n)(y - 2h) \right], \quad \gamma_n = \sqrt{1 - (\beta_n / kd)^2}, \]

\[ \psi_n(x) = w_n / (-ikd) \beta_n \sin(x / d + \beta_n / 2) + \sin(x / d + \beta_n / 2), \]  

where \( q=0 \) or \( q=1 \) for the cases in Fig.1a and Fig.1b, correspondingly, \( A_k(\gamma) = (w_k\gamma - 1)/(w_k\gamma + 1) \). \( \beta_n \) are eigenvalues satisfying to a transcendental complex equation. Algorithms for solution of complex transcendental equations with many roots determining eigenvalues for impedance waveguide is developed in this work. Using orthogonality of the eigenfunctions one can reduce Eq. (1) to an infinite set of algebraic equations

\[ A_n + \sum_{m=1}^{\infty} A_n U_{nn} = R_n, \quad n=1,2,3,... \]

The matrix coefficients are expressed as integrals of some special functions. The far field of the scattering is expressed using the bistatic diagram of scattering \( g(\alpha_0, \alpha_1) \)

\[ p_b \approx \frac{2}{(kdR_0)} \exp(ikd - i4)(\alpha_0, \alpha_1). \]  

Amplitudes of the back scattering (Fig.2) depend on impedances of the baffles and the walls. For sound absorbing baffle (the curves with vanishing reflection coefficients) the amplitude significantly decreases at normal incidence of the wave but retain big values at grazing incidence. The developed algorithms and programs give possibility to calculate bistatic diagrams, form-functions, acoustic power of the waves penetrating into waveguides and directivity properties of acoustical arrays situated inside deepenings in the baffles.

REFERENCES