A New Time-Domain Approach for Nonlinear Wave Propagation: Comparison With the KZK Equation Approach in the Case of Unfocused CW Beams

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Abstract: Using a newly-developed nonlinear wave propagation model, the acoustic pressure field of a plane circular transducer was simulated in water for a large range of CW-mode input excitation levels. The axial and lateral pressure profiles and their corresponding harmonic components were calculated using our model and the KZK model, and very good agreement was obtained between the two models. Furthermore, these results were verified by previously-published experimental measurements.

INTRODUCTION

We have already presented a time-domain numerical model for simulating acoustic pulse focusing in soft tissue. In this model, the main effects responsible for finite-amplitude wave propagation, i.e. diffraction, frequency-dependent attenuation and nonlinearity, were taken into account (1), (2). In the present work, a comparison between the results of our model with those of the KZK model in the case of a monochromatically-excited plane piston source is presented.

MODEL DESCRIPTION

In the general wave theory, differential equation of the evolution type is widely used:

$$\frac{\partial v(r, z)}{\partial z} = L \cdot v$$

(1)

where $v$ is particle velocity, $z$ is the coordinate in the direction of wave propagation, $r = t - z/c_0$ is retarded time and $c_0$ is the infinitesimal wave speed. Operator $L$ accounts for effects changing the waveform. Note that $L$ is a fairly complex integro-differential operator. The evolution equation (1) is associated with only one effect, whereas in reality the waveform is distorted due to several effects. In the presence of multiple effects, if each of them is fairly weak, the evolution equation can be derived simply by summing the corresponding operators from one-effect equations. The evolution equation we use for development of our model by taking into account the effects of absorption, nonlinearity and diffraction can therefore be written as:

$$\frac{\partial v}{\partial z} = L \cdot v = L_1 \cdot v + L_2 \cdot v + L_3 \cdot v$$

(2)

One of the possible ways to solve equation (2) numerically is to calculate the different effects independently using an operator-splitting algorithm. This algorithm is used in the numerical simulation of many physical problems, especially in optics (3). In this procedure, the propagation distance $z$ is divided into subintervals $\Delta z$. At each step $\Delta z$, the solution of a combined evolution equation is obtained by solving separate equations for each effect. This algorithm has been shown to be valid, when the subintervals $\Delta z$ are small enough such that the rate of change of the field variables is very small. It has been also shown that the operator-splitting method is independent of the order in which the effects are applied when the subintervals are small enough (1). In our model, the acoustic wave propagates plane-by-plane, from the surface of an ultrasonic source. In each plane, the effects of diffraction, absorption and nonlinearity are applied independently, by means of an operator-splitting algorithm, in order to calculate the pressure and normal particle velocity fields. In our simulations, a second-order accuracy operator-splitting algorithm has been used. To solve the diffraction equation, we used the Rayleigh integral (4). Using this integral, we calculated the normal particle velocity and the pressure field at each point of an observing plane. The absorption equation was solved by a minimum-phase digital filter model (5). Verification of the results obtained by applying this attenuator filter to a monochromatic sinusoidal wave demonstrated that it is possible to simulate exactly the frequency-dependent attenuation of soft tissues (1). Finally, using an analytical solution (Poisson solution) of the nonlinearity equation, we simulated the nonlinear wave distortion during its propagation in the medium (1).

COMPARISON WITH THE KZK MODEL

Using our model the acoustic pressure field generated by a pulsed focused intensive source was simulated and very good agreement between the theoretical and experimental pressure-time waveforms was obtained. These results have already been presented (1), (2). Here, a comparison between the results of our model with those already obtained by the KZK model in the
case of an unfocused source with CW excitation is presented. The KZK results we use in this study are based on a numerical implementation developed by Naze Tjotta and Tjotta (6) and experimentally verified by Nachef et al. (7). The source, input excitation and medium parameters used both in our and the KZK model are as follows:

Source parameters: PZT plane circular transducer with an effective radius of 23.2 mm.

Input excitation parameters: 10-μs-long 1 MHz sinusoidal tone burst with three different pressure amplitudes on the source surface i.e. 2, 6.4 and 14.3 bars (1 bar = 100 kPa).

Medium parameters: Degassed and deionized water with infinitesimal sound speed $c_0 = 1500 \text{ m/s}$, density $\rho = 1000 \text{ kg/m}^3$, coefficient of attenuation $\alpha = 0.0022 \text{ dB/cm MHz}^2$ and parameter of nonlinearity $\beta = 1 + B/2A = 3.5$.

The axial and lateral acoustic pressure amplitude profiles of the first five harmonics were calculated by our model. The calculations were performed for three different source surface pressure amplitudes. A qualitative comparison between these profiles and those already obtained by the KZK model (7, Figs. 13-15) shows very good agreement between the results of the two models. More quantitative analysis is in progress at present. In Fig. 1 the calculated profiles of acoustic pressure amplitude for the fundamental $(n=1)$, third $(n=3)$ and fifth harmonics $(n=5)$ are shown.

![FIGURE 1. Axial (upper figures) and lateral (lower figures) profiles of acoustic pressure amplitude for the fundamental $(n=1)$, third $(n=3)$ and fifth harmonics $(n=5)$. Solid, dashed and dotted lines correspond to the input pressures of 14.3, 6.4 and 2.0 bars respectively. Vertical axes represent the pressure amplitude in bar and horizontal axes represent distance in mm.](image)

At the present, all implementations of the KZK model are limited to planar or weakly-focused sources. This study, along with our previous work, reveals the validity of our finite-amplitude wave propagation model for a wide degree of source focusing (from a planar to a highly-focused source), and in CW as well as PW modes.

REFERENCES


