A tubular piezoelectric actuator and its characteristic analysis by finite element modelling

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Abstract: The actuator of tubular configuration is proposed for the three-dimensional movement at the tip. A cylindrical shell made of piezoceramic, polarized in radial direction, with several pairs of electrodes provided on both surfaces of the shell is considered. The displacement at the tip of the shell is expressed in terms of the linear combination of the electrical input voltages applied to each pair of the electrodes. The combination of the electric voltages properly chosen can control the movement. The proportional coefficients of their terms are to be determined by the experiment or numerical simulation. Here, the determination is made based on the finite element modelling.

PRINCIPLE

An actuator that provides the capability of the three-dimensional movement but with small displacement is sometimes required. Here we propose the actuator of the configuration as shown in Figure 1. It is a cylindrical shell made of piezoelectric ceramics polarized in radial direction. The inner surface is fully electrode, which is used as the ground. On the outer surface, four electrodes are partially provided to which electric potentials are properly applied for the excitation. They are named x1, x2, y1 and y2.

If positive and negative voltages are given to the electrodes x1 and x2 respectively, the shell under the electrode x1 becomes shorter while one under the x2 becomes longer. So the tip of the actuator moves toward –x direction. The same effect occurs about y direction. If positive voltages of the same amplitude are given to all of the electrodes, the tip of the shell moves toward –z direction. The relation of this action can be expressed as follows:

\[
\begin{bmatrix}
  u_x \\
  u_y \\
  u_z 
\end{bmatrix} = \begin{bmatrix}
  -\alpha & \alpha & 0 & 0 \\
  0 & 0 & -\alpha & \alpha \\
  -\beta & -\beta & -\beta & -\beta
\end{bmatrix} \begin{bmatrix}
  V_{x1} \\
  V_{x2} \\
  V_{y1} \\
  V_{y2}
\end{bmatrix}
\]

(1)

where \( V_{x1} \) to \( V_{y2} \) and \( u_x \) to \( u_y \) indicate the voltages applied to the electrodes x1 to y2, and the displacement at the tip of the shell. \( \alpha \) and \( \beta \) are the proportional coefficients to be determined by the experiment or the numerical analysis.

FINITE ELEMENT MODELLING

The shell is divided into hexahedral elements. The discretized equations of the movement are expressed for the system as follows:

\[
\begin{bmatrix}
  K & \Gamma \\
  \Gamma^T & G
\end{bmatrix} \begin{bmatrix}
  u \\
  \phi
\end{bmatrix} = \begin{bmatrix}
  F \\
  Q
\end{bmatrix}
\]

(2)

\( K \): stiffness matrix  \( \Gamma \): electro-mechanical coupling matrix  \( G \): capacitance matrix  \( u \): displacement vector  \( \phi \): electric potential vector  \( F \): force vector  \( Q \): charge vector

The finite element division is chosen to be \((r : \theta : z) = (1 : 56 : 30)\) as illustrated in Figure 2. It consists of 1,680 elements and requires 11,313 degrees of freedom or dimensions. Electric potential near the edge of the electrodes varies sharply, where the region is divided into smaller elements. The mechanical boundary condition is assumed to be fixed at the nodes on the bottom surface \((z = 0)\). The electrical potential is assumed to be ±1 V or 0 V applied to the electrodes, in which the sense is chosen depending on the purpose.
NUMERICAL DEMONSTRATION

For numerical analysis, the code which was previously developed is modified to include the static movement[1]. In this code, an isoparametric cubic element with eight nodes is used[2]. Displacements are defined, for the x, y and z direction at each node, which are linearly interpolated. The effect of the second order polynomial terms is also considered without increasing the number of the nodes. That is, although nine unknowns must additionally be defined to interpolate the displacements with the situation, excess unknowns are eliminated by imposing the equivalence[3]. Electric potential is linearly interpolated.

This actuator is thought to be used in very low frequency near the static deformation. The material constants of piezoelectric ceramic assumed are shown in Table 1.

Figure 3 illustrates calculated displacements of the actuator in each case. The displacements illustrated is $3 \times 10^9$ times magnified.

The tip displacements in the cases of (a), (b) and (c) are $(x, y, z) = (-6.24 \times 10^{-9}, 0, 0), (-0, 6.24 \times 10^{-9}, 0)$ and $(0, 0, -8.36 \times 10^{-10})$ respectively. $\alpha$ and $\beta$ in equation (1) are determined to be $3.12 \times 10^{-9}$ and $2.09 \times 10^{-10}$. The values obtained reasonably correspond to the measured ones.

**TABLE 1. Material constants of piezoelectric ceramic**

<table>
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<tr>
<th>$c_{33}$</th>
<th>$c_{33}^E$</th>
<th>$c_{13}$</th>
<th>$c_{33}^E$</th>
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<td>9.1</td>
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<td>2.5</td>
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<tr>
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<td>$\varepsilon_{33}^S$</td>
<td>$\varepsilon_{33}^E$</td>
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<tr>
<td>904</td>
<td>904</td>
<td>818</td>
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</table>

$E_f: (\times 10^{10} \text{N/m}^2), \varepsilon: (\text{C/m}^2)$

**REFERENCES**


**FIGURE 2. Finite element division for actuator.**

(a) $V_{x1} = 1V$ and $V_{x2} = -1V$. (b) $V_{y1} = 1V$ and $V_{y2} = -1V$. (c) $V_{x1} = V_{x2} = V_{y1} = V_{y2} = 1V$.

**FIGURE 3. Displacement of actuator in each case.**

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