The Condition for Beam-bending Modes to Dominate in the Vibro-acoustic Behaviour of a Circular Cylindrical Shell

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Abstract: An analysis of the basic vibration behaviour of circular cylindrical shells shows that for an infinite length cylindrical shell, the beam-bending wave would always be propagating with other flexural waves of different circumferential mode numbers. However, for a finite length circular cylindrical shell, it is found that if the length of the shell is much greater than a value which depends on the radius and the thickness of the shell, below the cut-off frequency of \( n=2 \) mode, beam-bending modes would dominate the vibration response so that it can be treated as a beam with reasonable accuracy.

INTRODUCTION

The acoustic radiation from circular cylindrical shells is of fundamental and applied interest primarily because cylindrical shells are widely used in industries. However, according to previous studies, only few special cases, for example, a cylindrical shell under the assumption of beam-bending, has a simple analytical solution[1]. Obviously, in practice, the vibration behaviour of a cylindrical shell may not be assumed to be beam-bending in the whole frequency range of interest because the vibration behaviour of a cylindrical shell changes with frequency. Normally, it is thought that for a relatively long circular cylindrical shell at low frequencies, it can be treated as a beam to facilitate the analysis. However, it is not clear yet how long a cylindrical shell has to be before it can be considered as a beam to achieve acceptable accuracy. Therefore, for acoustics purpose, bending modes which correspond to \( n=1 \) of the shell are important and warrant further analysis.

The objective of this study is to determine the condition under which the vibration and acoustic behaviour of a finite length circular cylindrical shell can be treated as a beam.

DISCUSSIONS ON THE BENDING WAVES AND BENDING MODES

As discussed in [2], for an infinite length circular cylindrical shell, in the axial direction there are a series of flexural waves corresponding to different circumferential modes, in which the wave of \( n=1 \) is the bending wave. Each of these waves has its own cut-off frequency, above which the corresponding wave can occur. As these cut-off frequencies \( f_{vcn}^2 \) actually correspond to the natural frequencies of the modes with zero axial wavenumber for free-free cylindrical shells, a simple expression[3] can be used here.

\[
f_{vcn}^2 = \frac{h^2}{12a^2} \frac{n^2(n^2-1)^2}{(n^2+1)^2} f_r^2
\]

where \( a, h \) are the radius and thickness of the shell respectively, and \( f_r \) is the ring frequency of the shell. It can be seen that as the cut-off frequencies for mode \( n=0 \) and \( n=1 \) are all zero, \( n=0 \) modes would always occur in addition to bending modes even at low frequencies. However, for a given frequency, as the axial wavenumber of mode \( n=0 \) is much smaller than that of mode \( n=1 \)[3], the effect of mode \( n=0 \) might be small so that bending waves would dominate the behaviour of the cylindrical shell. Actually in [2], it was shown that below the cut-off frequency of \( n=2 \), the point input impedance of an infinite length cylindrical shell takes the form of an equivalent beam in bending motion. This result shows that for an infinite length cylindrical shell, below \( f_{vc2} \), it can be simply treated as a beam in the corresponding vibration and acoustics analysis.

For finite length cylindrical shells, according to the vibration theory, the vibration response of the shell is the superposition of all the vibration modes. At frequencies well below the first natural frequency of the shell, the vibration response is dominated by the behaviour of the first mode. Therefore, it can be expected that for a finite length cylindrical shell with both ends free, if there is no vibration mode below \( f_{vc2} \), the behaviour of the shell at low frequencies is mostly dependent on the behaviour of the mode \( n=2 \). Obviously in this case, the cylindrical shell cannot be treated as a beam. Only when the cylindrical shell is long enough that bending modes occur below \( f_{vc2} \),
could the cylindrical shell behave like a beam at low frequencies. Note that the wavenumber of the first bending mode in the axial direction for a clamped-clamped cylindrical shell can be approximately written as[3]
\[
k_z = 1.5n/\pi
\]  
(2)
The relationship between the frequency and wavenumber for bending wave is [2]:
\[
f = \frac{\sqrt{2}}{2} (k_z a)^2 f_r
\]  
(3)
By using equations (1), (2) and (3), the condition for the natural frequency of the first bending mode being smaller than \( f_{vc2} \) for all possible boundary conditions is
\[
l > 1.5\pi a \sqrt{\frac{a}{h}} \sim S \sqrt{\frac{a}{h}} \quad S = 2\pi a
\]  
(4)
where \( l \) is the length of the cylindrical shell. Therefore, when equation (4) is satisfied, the low frequency behaviour of the shell would be dominated by the bending modes. As the length becomes longer, there would be more bending modes below \( f_{vc2} \), and the results obtained by the beam model would be more accurate.

SIMULATIONS AND CONCLUSIONS

To verify the above result, two groups of steel cylindrical shells of different radius/thickness ratios were investigated. By using an analytical expression[4], the radiation efficiencies of the shells with different lengths have been calculated and compared with Richards' results[1] in Fig.1. It can be seen that as the length of the shell increases, the radiation efficiencies of finite length cylindrical shells approach Richards' results. Also, for the case in which the cut-off frequency for \( n=2 \) is lower than the critical frequency, Richards' results only apply below \( f_{vc2} \). When \( f_{vc2} \) is greater than the critical frequency, Richards' results can be used for the whole frequency range. However, if the length of the shell doesn't satisfy equation (4), the shell cannot be treated as a beam.

Results obtained by the boundary element method also agree with the analytical results shown in Fig.1.

\[ \text{FIGURE 1. Radiation efficiencies of finite length cylindrical shells: (a), } a=0.042 \text{ m; } h=0.004 \text{ m; } f_{vc2}=1.4 \text{ kHz; } f_c=3 \]

\[ \text{kHz; } S \sqrt{\frac{a}{h}} =0.8 \text{ m, (b), } a=0.0195 \text{ m; } h=0.003 \text{ m; } f_{vc2}=5 \text{ kHz; } f_c=4 \text{ kHz; } S \sqrt{\frac{a}{h}} =0.3 \text{ m} \]

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