Acoustic wave scattering from a coated cylindrical shell

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Abstract: This work presents the analysis of a theoretical model for evaluating the effect of a coating (covered on a cylindrical steel shell) on directivity patterns. Directivity patterns computed for a coated cylindrical shell are compared with those computed for a rigid cylinder to observe the difference between two results. Directivity patterns were calculated from the farfield pressures scattered from the two different cylindrical geometries.

INTRODUCTION

A theoretical model was developed to evaluate the effect of a coating on a cylindrical shell. The coating is designed to reduce the flexural wave noise generated by the vibration of the cylindrical shell. In underwater applications, this coating is called an inner decoupler. The outer surface of the composite structure, which consists of the cylindrical shell and the inner decoupler, is in contact with water; the core (cavity) of the structure contains air. The analysis of the problem is made for a double-layer cylindrical structure that is immersed in water and excited by an incident acoustic wave. The pressure field scattered from a solid cylinder has been studied by Morse and Bowman et al. A double layer problem similar to the present study was investigated by Gaunaurd and Flax and Neubauer. The theory of elasticity, the elastic wave equation, the acoustic wave equation, and pertinent boundary conditions are used in formulating the problem. The two-dimensional problem is treated in this study. This implies that all the quantities are independent of the axial coordinate. Numerically calculated results for both coated and uncoated cylindrical shells are presented to compare these two directivity patterns.

FORMULATION OF THE PROBLEM AND NUMERICAL RESULTS

The geometry of the theoretical model is depicted in Fig. 1. The outer surface of the composite structure is in contact with water and the core of the structure contains air. The major analysis of this study is to formulate the scattering pressure field from the two-layer structure. The pressure fields in the surrounding fluid medium and the core region are governed by the acoustic wave equations. The propagation of the wave in the cylindrical shell and the inner decoupler is described by the vector differential equation that governs the small elastic motion in the elastic medium.

The boundary conditions to be satisfied at the interface between the outer surface of the composite structure and the surrounding fluid medium, i.e., \( r=R_0 \), are written as

\[
\begin{align*}
\sigma^{(n)}_{rr,1} \big|_{r=R_0} &= -p_0^{(n)} \big|_{r=R_0}, \\
\frac{\partial^2 u^{(n)}_{r,1}}{\partial t^2} \bigg|_{r=R_0} &= -\frac{1}{\rho_0} \frac{\partial p_0^{(n)}}{\partial r} \bigg|_{r=R_0}, \quad \sigma^{(n)}_{rr,1} \big|_{r=R_0} = 0.
\end{align*}
\]

The boundary conditions at the interface between the cylindrical shell and the inner decoupler, i.e., \( r=R_1 \), are written as

\[
\begin{align*}
\sigma^{(n)}_{rr,1} \big|_{r=R_1} &= \sigma^{(n)}_{rr,2} \big|_{r=R_1}, \\
\sigma^{(n)}_{r\theta,1} \big|_{r=R_1} &= \sigma^{(n)}_{r\theta,2} \big|_{r=R_1}, \\
u^{(n)}_{r,1} \big|_{r=R_1} &= u^{(n)}_{r,2} \big|_{r=R_1}, \\
u^{(n)}_{\theta,1} \big|_{r=R_1} &= u^{(n)}_{\theta,2} \big|_{r=R_1}.
\end{align*}
\]

The boundary conditions to be satisfied at the interface between the inner surface of the composite structure and the core fluid, i.e., \( r=R_2 \), are written as

\[
\begin{align*}
\sigma^{(n)}_{rr,2} \big|_{r=R_2} &= -p_3^{(n)} \big|_{r=R_2}, \\
\frac{\partial^2 u^{(n)}_{r,2}}{\partial t^2} \bigg|_{r=R_2} &= -\frac{1}{\rho_3} \frac{\partial p_3^{(n)}}{\partial r} \bigg|_{r=R_2}, \quad \sigma^{(n)}_{rr,2} \big|_{r=R_2} = 0.
\end{align*}
\]

It is important to mention the meanings of the superscripts and subscripts associated with a certain quantity. The radial stress associated with the normal mode (n) of the inner decoupler (layer 1) acting on the
interface r=R_0 is expressed as \[ \varepsilon^{(n)}_{\text{tr},1} \] \[ \tau_{\text{tr},1} \] \[ \tau_{R_0} \]. The displacements of the composite structure and acoustic pressures in the exterior and interior fluid media are denoted by u and p with respective subscripts and superscripts. Using the above boundary conditions, one may obtain a system of linear algebraic equations (10x10) to be solved for the unknown coefficients that will be used for the necessary expressions. The normalized farfield pressure scattered from the composite structure (beam pattern) as a function of the azimuthal angle, \( \theta \), is expressed as:

\[
P(\theta) = 20 \log_{10} \left( \frac{\Phi(\theta)}{\Phi(\theta)_{\text{max}}} \right) \text{ dB}, \quad \Phi(\theta) = \sum_{n=0}^{\infty} \varepsilon_n A^{(n)}_0 \cos(n\theta).
\]

Note that \( \varepsilon_n = 1 \) (n=0) and \( \varepsilon_n = 2 \) (n>0); and the coefficient, \( A^{(n)}_0 \), has been obtained from the 10x10 algebraic equations. Figure 2 presents a comparison between the directivity pattern for the cylindrical shell with a thin inner decoupler (solid line) and that for the rigid cylinder (dotted line), with kR=10. Figure 3 presents the directivity patterns computed at 400 Hz for the cylindrical shell with \( h_1 = 2.54 \text{ cm} \) (solid line) and an equivalent rigid cylinder (dotted line). Figure 4 presents the similar results calculated at 4200 Hz. At a low frequency (400 Hz), the coating makes the beamwidth for the coated cylindrical shell narrower than that for the solid cylinder, as shown in Fig. 3. However, at a high frequency (4200 Hz), the coating makes the beamwidth for the coated cylindrical shell much wider than that for the solid cylinder, as shown in Fig. 4. The main text of this work will present various parametrical studies such as the effects of the configuration of the two-layer structure and material properties of the inner decoupler.

**REFERENCES**