Resonant Frequencies into Degeneration of Hybrid Longitudinal and Torsional Vibration System

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Abstract--The Langivan vibrator is made of the longitudinal(L) and torsional(T) piezoelectric plates, which can be used for a transducer of an ultrasonic motor and ultrasonic welding. The L and T sound velocities are different. To make resonant frequencies of L and T modes into degeneration, one of the methods is to connect a matching block with a thin neck to the end of the vibrator. Two calculation methods-- one dimensional network transmission matrix and FEM have been used and compared. The L and T resonant frequencies of several steel bars with variant thin necks and matching blocks were calculated. Some experiment results of measuring the resonant frequencies have been compared with the calculation. Six prototypes of hybrid L-T ultrasonic motor have been fabricated, and they can be operated very well.

Introduction

The vibration systems of the L-T composition mode are interested in the field of the ultrasonic motor and ultrasonic welding. Because the L-T mode-conversion motors have high power, high efficiency, and are cheaper, they are studied up to now(1-3). However it is difficult to reverse rotation and to control. The other vibration systems of L and T hybrid(4,5)have been investigated also. The L and T mode can be adjusted respectively in it, and can output higher power. It is necessary to make L and T resonant frequencies into degeneration for high power and efficiency in practice ultrasonic motors.

We use the steel bars as the sample to study the L and T resonant frequencies into degeneration. To connect a match block with thin neck is a good method. Three sorts--six prototypes of L and T hybrid ultrasonic motor with matching block have been fabricated.

Resonant frequency of L-T vibration system

The resonant frequencies for the steel bar shown in Fig 1 have been calculated by network method and FEM. The four-terminal network matrix of the L-mode with uniform section is following:

\[
\begin{bmatrix}
\alpha_{11} & \alpha_{12} \\
\alpha_{21} & \alpha_{22}
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix}
\]

where \(\alpha_{11} = \cos k_l, \alpha_{12} = -j \sin k_l, \rho c_s, \alpha_{21} = -j \rho c_s \sin k_l, \alpha_{22} = \cos k_l\)

\(\rho\): density, \(c\): the phase velocity of L-mode, \(k = \omega / c, S\): area.

The four-terminal network matrix of the T-mode with uniform section is following:

\[
\begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix}
\begin{bmatrix}
\alpha_{11} & \alpha_{12} \\
\alpha_{21} & \alpha_{22}
\end{bmatrix}
\begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix}
\]

\(\alpha_{11} = \cos k_t, \alpha_{12} = -j \sin k_t, \rho I / C_t, \alpha_{21} = -j \rho I / C_t \sin k_l, \alpha_{22} = \cos k_t\)

\(c_t\): the phase velocity of T-mode, \(k_t = \omega / c, I_p = \pi r^4 / 2\): moment of inertia, \(r\): the radius of the bar.

The transmission matrix for a vibration system with different section bar is:

\[
\begin{bmatrix}
\bar{\alpha}_{11} & \bar{\alpha}_{12} \\
\bar{\alpha}_{21} & \bar{\alpha}_{22}
\end{bmatrix}
\begin{bmatrix}
\alpha_{11} & \alpha_{12} \\
\alpha_{21} & \alpha_{22}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\alpha_{11} & \alpha_{12} \\
\alpha_{21} & \alpha_{22}
\end{bmatrix}
\begin{bmatrix}
\alpha_{11} & \alpha_{12} \\
\alpha_{21} & \alpha_{22}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\bar{l}_1 \\
\bar{l}_2 \\
\bar{l}_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix}
\]

Fig 1. the construction of a steel bar

Fig 2. The distribution of T-mode displacement
When a vibration system is operated in free condition, $\alpha_{z_1} = 0$ and $\alpha_{z_1}^n = 0$, the resonant frequencies of L-mode and T-mode can be obtained from it. The thin neck takes on isolation T-mode as shown in Fig.2, the neck is thinner, the isolation is better. The resonant frequencies $f_L$ and $f_T$ change with $D_2$ and $l_3$. The Fig.3 shows the resonant frequencies $f_L$ and $f_T$ against $l_3/l_1$ with different $D_2/D_1$.

![Fig3](image3.png)

![Fig4](image4.png)

**Fig.3.** $f_L$ and $f_T$ change with $l_3/l_1$ and $D_2/D_1$.

**Fig.4.** T-mode of stator calculated by FEM

### Table 1. $f_L$ and $f_T$ of steel bar with different neck ($D_1 = D_3 = 30\text{mm}, l_1 = 60\text{mm}, l_2 = 2\text{mm}, l_3 = 38\text{mm}$)

<table>
<thead>
<tr>
<th></th>
<th>$D_2$ [mm]</th>
<th>10</th>
<th>16</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_L$ kHz</td>
<td>Network</td>
<td>22.35</td>
<td>24.4</td>
<td>24.98</td>
<td>25.50</td>
</tr>
<tr>
<td></td>
<td>FEM</td>
<td>16.47</td>
<td>20.55</td>
<td>22.94</td>
<td>25.53</td>
</tr>
<tr>
<td></td>
<td>Experiment</td>
<td>15.95</td>
<td>21.60</td>
<td>24.32</td>
<td>25.58</td>
</tr>
<tr>
<td>$f_T$ kHz</td>
<td>Network</td>
<td>26.71</td>
<td>29.21</td>
<td>30.36</td>
<td>31.00</td>
</tr>
<tr>
<td></td>
<td>FEM</td>
<td>26.32</td>
<td>27.83</td>
<td>29.34</td>
<td>31.04</td>
</tr>
<tr>
<td></td>
<td>Experiment</td>
<td>27.09</td>
<td>28.50</td>
<td>31.40</td>
<td>32.20</td>
</tr>
</tbody>
</table>

The table-1 shows that the calculated results are consistent with experiment basically, and we notice that the calculated $f_L$ by network method has bigger deviation with experiment at $D_2 = 10\text{mm}$.

The three sorts and six prototypes of ultrasonic motors have been fabricated by use of L and T resonant frequencies into degeneration. For one, it is shown in Fig.4. The stator size is: $D_1 = 12.6\text{mm}, l_1 = 25\text{mm}$, $D_2 = 6.6\text{mm}, l_2 = 1.2\text{mm}$, $D_3 = 12.6\text{mm}, l_3 = 10\text{mm}$. $f_L$ = 51.9kHz, $f_T$ = 52.5kHz. It can operate at 20-200rpm and the maximum torque is 0.05Nm. The preload makes the $f_L$ lower slight and $f_T$ higher slight.

It has been analyzed that the feasibility of frequencies tuning through a matching block, and summarized how to affect the resonant frequency by the diameter and thickness of the thin neck and matching block. The calculation results of FEM are better than one dimensional network transmission matrix method.

### Reference


