On the Theory of Supercompression of a Gas Bubble in a Liquid-Filled Flask


Ufa Branch of Russian Academy of Sciences, K. Marx Str. 6, Ufa, 450000, RUSSIA
* Rensselaer Polytechnic Institute, Troy, NY, 12180-3590, USA

Abstract: The spherically-symmetric problem is considered on gas bubble in the center of a spherical flask filled with compressible liquid that is exited by pressure oscillations on the flask wall. A generalization of the Rayleigh-Plesset equation for compressible liquid is given in the form of two ordinary difference-differential equations that take into account the pressure waves reflected from the bubble and those that are incident on the bubble from the flask wall.

An approximate theory has been erected by Prosperetti and Lezzi (1) regarding radial motions of a spherical bubble in an infinite compressible liquid. A whole family of equations of bubble oscillations has been obtained including other authors' equations as specific cases. All these equations are shown to be equivalent, as they have one and the same order of accuracy by Mach number. In deriving the equations, the statement is used that bubble oscillations do not affect the outer acoustic field of pressure at infinity. The fact that the fluid is unbounded permits to consider the problem on bubble oscillations separately from the acoustic problem taking into account the outer acoustic field as a definite motive force for the bubble.

The present paper deals with an associated problem on oscillations of a limited liquid volume and of a gas bubble, in which the flask wall is used to excite the liquid. The spherically-symmetric radial flow of a compressible liquid in a spherical flask (with radius $R$) around a small spherical gas bubble (with radius $a$) located at the center of the flask is considered. For the low Mach number stage of the process ($M_a = \dot{a}/C \ll 1$) the space between the bubble interface and the flask wall consists of three zones:

1. The external zone, where the weak compressibility of the liquid is essential but convective displacements of the liquid are small. In this region the liquid motion has an acoustic wave propagation character.
2. The internal zone, near to the bubble interface, where liquid compressibility has negligible influence on the process and the motion occurs only because of the compression and expansion of the bubble.
3. The intermediate zone where both liquid compressibility and non-linear inertia forces (due to the convective accelerations) are important.

In order to obtain an overall solution it is necessary that the asymptotic solutions of the external and internal zones be matched in the intermediate zone by the compatibility conditions for the volume flow and pressure. It leads to a long-wave approximation of the equation for bubble oscillations in a compressible liquid (2):

$$
\left(1 - \frac{\dot{a}}{C}\right) a \ddot{a} + \frac{3}{2} \left(1 - \frac{\dot{a}}{3C}\right) \dot{a}^2 = \left(1 - \frac{\dot{a}}{C}\right) \frac{p_a - p_l}{\rho} + \frac{a}{C} \frac{d}{dt} \left[ \frac{p_a - p_l}{\rho} \right],
$$

$$
p_a = p_g(a) - \frac{2\sigma}{a} - \frac{4\mu \dot{a}}{a}, \quad p_l = p_0 - \frac{2\rho}{C} \psi_2,
$$

$$
p_R(t) = p_0 - \frac{\rho}{R} \left[ \psi_2 \left(t + \frac{R}{C}\right) - \psi_2 \left(t - \frac{R}{C}\right) - \dot{Q} \left(t - \frac{R}{C}\right) \right].
$$

This system of ordinary difference-differential equations having both lagging (retarding) and leading potentials is closed for a given density $\rho$, sound speed $C$, surface tension $\sigma$, viscosity $\mu$, of the liquid, equation of state for the bubble gas $p_g(a)$, and pressure on the flask wall $p_R(t)$. Here $\psi_2$ is the velocity potential of the wave incident on the bubble from the flask wall.

The problem is, to calculate the evolution of the bubble radius $a(t)$ knowing the evolution of the flask's wall pressure $p_R(t)$. In general case the solution of the problem may be obtained by the numerical integration...
of the partial differential equations. In the paper by Moss et al. (3) a numerical code for partial differential
equations was used for the calculation of the bubble behaviour not only for the bubble implosion stage, but
for the long wave length and low Mach number stages as well. Needless to say, the computer run time was
very large. In the Moss's work, the direct problem was solved for a sinusoidal pressure change on the flask:
\[ p_R(t) = p_0 - \Delta p_R \sin \omega t, \quad t > 0. \]

The bubble radius evolution \( a(t) \) was calculated but for the first oscillation and not for the periodic regime, which takes place only after many oscillations have concealed the initial conditions. Significantly, this is what is measured in experiments.

In Fig. 1 the results of our calculations are presented. The first oscillation agrees with the Moss et al.
(3) calculation (dotted line). However, one can see that the second oscillation differs essentially from the
first, the third differs from the second, and only after many oscillations (\( t \gg \omega^{-1} \)) a periodic regime does
take place.

**FIGURE 1.** Excitation of bubble (\( a_0 = 10 \mu m \)) oscillations in a flask (\( R = 5 cm \)) filled with water (\( p_0 = 1 \) bar, \( T_0 = 300 \) K) when the flask wall produces sinusoidal pressure oscillations with the amplitude, \( \Delta p_R = 0.25 \) bar, and the
frequency, \( f = 45 \) kHz, which is equal to a flask acoustic resonance frequency. On the right side the ideal periodic regime is shown which corresponds to that resonant case.

Linear and non-linear periodic bubble oscillations are analyzed analytically. Non-linear resonant and
near-resonant solutions are obtained for the bubble nonharmonic oscillations excited by harmonic pressure
oscillations on the flask wall. The influence of heat transfer phenomenon on the bubble oscillations is
analysed.

**REFERENCES**

