Experimental identification of mechanical joint properties

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Abstract: In this paper, a hybrid method based on the relation between components modeling and assembly FRF matrices is proposed for identifying joint parameters of assembled structures. Components FRF matrices are calculated by FEM, so FRFs have just to be measured on assembly. Numerical tests on a simple discrete mechanical system show how sensitivity can be used to choose optimum measured d.o.f.s and frequency range and to minimize errors introduced by noisy FRF values in results.

INTRODUCTION

The prediction of the vibroacoustic behavior of complexe structures composed with many substructures is often altered by poor modelling of junctions between the different substructures. Perfectly rigid or flexible junctions are actually not realistic. Moreover some junctions like screwed, stucked or welded ones, which cannot be isolated, have to be characterised only within the whole structure. Global fitting methods which permit an adjustment of global matrices can be separated from local methods which attempt to find optimum values of the parameters. The local methods are more easily subject to physical interpretation. Except particular cases where a simple relation between unknown parameters and some easily measurable indicators exists, general techniques use an iterative optimisation process with an objective function equal to the difference between measured and calculated indicators like eigenvalues, eigenvectors or Frequency Response Functions [1]. Tsai [2] proposed an expression relying substructures FRF matrices and assembly FRF matrices which is used to obtain directly an estimation of the stiffness and damping matrices of the junction. In his paper [2], all matrices have to be measured.

THEORY

Let us consider a structure made up of two substructures '1' and '2' linked through a massless viscoelastic joint described by its stiffness matrix, \( K \), and damping matrix, \( C \). Sub-dividing the substructures d.o.f.s in terms of \( n_{\text{int}} \) internal d.o.f.s, referred to by superscript 'int' and \( n_{\text{int/jmn}} \) junction d.o.f.s, referred to by superscript 'jmn', it can be shown that (see Tsai [2]):

\[
\begin{bmatrix}
U_{1,\text{int}}^t \\
U_{2,\text{int}}^t
\end{bmatrix}
= H_3
\begin{bmatrix}
P_{1,\text{int}}^t \\
P_{2,\text{int}}^t
\end{bmatrix}
\]

with

\[
H_3 = \begin{bmatrix}
H_{1,\text{int/int}} & 0 \\
0 & H_{2,\text{int/int}}
\end{bmatrix}
+ \begin{bmatrix}
-H_{1,\text{int/jmn}} & 0 \\
0 & H_{2,\text{int/jmn}}
\end{bmatrix}
H_{\text{coup}}
\begin{bmatrix}
H_{1,\text{jmn/int}} & -H_{2,\text{jmn/int}}
\end{bmatrix}
\]

(2)

and

\[
H_{\text{coup}} = \left( (K + j\omega C)^{-1} + \begin{bmatrix}
H_{1,\text{jmn/jmn}} & H_{2,\text{jmn/jmn}}
\end{bmatrix}^{-1}\right)^{-1}
\]

where \( H_{i,j} \) is the receptance matrix of substructure \( i, i=1,2 \), wherein the superscripts 'x,y' denote the involved d.o.f.s and \( F_i \) denote the applied loads.

By taking \( n_{\text{jmn}} \) internal d.o.f.s, all matrices of equation (2) are square and can be inverted to obtain the following equation:

\[
\begin{bmatrix}
-H_{1,\text{int/jmn}} \\
H_{2,\text{int/jmn}}
\end{bmatrix}(K + j\omega C)\begin{bmatrix}
H_{1,\text{jmn/int}} \\
-H_{2,\text{jmn/int}}
\end{bmatrix} = H_3
\]

(3)
After calculating substructures FRF matrices the coefficients of matrices $K$ and $C$ can be identified at each frequency by measuring assembly matrix $H_3$ and resolving the square linear system (3). But a lack of information (noisy $H_3$ values) can lead to ill-conditioned matrices and in general cases more equations than unknowns are needed. To diminish the number of unknowns it is possible to fix the a priori known coefficients in $K$ and $C$. To increase the number of equations, the joint matrices can be supposed constant on a frequency range. Finally, a rectangular linear system can be obtained and the least square method can be used to identify the unknown parameters. Differentiation of equation (2) leads directly to the expression of assembly matrix sensitivity to joint coefficients. So optimum measured FRFs and frequency range can be chosen.

**NUMERICAL TESTS**

For all tests FRF matrices are calculated by FEM. Residual flexibility is added to FRFs at junction d.o.fs to prevent the modal truncature from introducing errors in the coupling process. Gaussian type noise has also been added to assembly FRF’s magnitude and phase. The percentage of noise is the ratio of RMS value of noise to the RMS value of the signal corresponding to the FRF. The only results presented concern the system described Fig.1 and already used in Ref. [3]. Joint parameters are $k_7, k_8, c_7$ and $c_6$. Internal d.o.fs corresponding to lumped masses $m_1$ and $m_6$ are used for the identification. When there is no added noise, the identified values are exactly the values entered in FEM model for every calculated frequency. When noise is added, identified values are correct only in the frequency range where assembly FRF matrix is sensitive to joint parameters. For global identification the results of a first identification on all calculated frequencies are used to calculate the sensitivity and select the frequency range for a second identification. The global identified parameters for noisy FRF values are given in Table 1.

![FIGURE 1. Simple discrete mechanical system tested.](image)

**TABLE 1.** Comparison of exact and identified values for the two bandwidths.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1 - 15 Hz</th>
<th>1% noise</th>
<th>6.9 - 8.4 Hz</th>
<th>1% noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_7$ (N.m$^{-1}$)</td>
<td>20000</td>
<td>15314</td>
<td>20010</td>
<td>5 ± 4</td>
</tr>
<tr>
<td>$k_8$ (N.m$^{-1}$)</td>
<td>12000</td>
<td>13646</td>
<td>12000</td>
<td>2 ± 2</td>
</tr>
<tr>
<td>$c_7$ (N.m$^{-1}$.s)</td>
<td>12.5</td>
<td>23.6</td>
<td>12.4</td>
<td>8 ± 3</td>
</tr>
<tr>
<td>$c_6$ (N.m$^{-1}$.s)</td>
<td>10</td>
<td>6.7</td>
<td>9.9</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**REFERENCES**