MICROMECHANICAL SYSTEMS
Acoustic Eyeballs with MEMS Retinas; Quantification of Crosstalk

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A microelectromechanical system is considered in which a rectangular array of acoustical sensors on a silicon "chip" functions in a manner analogous to the retina within an eyeball. Incident acoustic waves from any given direction pass through an acoustic lens and are focused on the plane of the array. Each array element is idealizable as a square hole in a slab of crystalline silicon with a rigid backing. "Cross-talk" between elements results because the surface is not a flat and smooth rigid surface. The size and profile of the incident focused beam at the nominal surface can be predicted by general acoustical wave theory and depends on the wavelength, the focal length, and the diameter of the lens. Given that the resulting spot is of the size of one of the holes, one seeks the ratio of the fluid velocity amplitude within the hole at which the beam is directed to that within an adjacent hole. The ratio can be comparable to unity when the elements are close together. Various theoretical techniques and computational techniques indicate how the design parameters of the system affect this ratio.

MEMS ARRAY AS A RETINA

A MEMS chip with a rectangular array of square holes, each hole intended to act as an acoustic sensor, is shown in Fig. 1. In the envisioned design, each hole is of finite depth and has a cross-sectional area that varies with depth distance. Within each hole, a specially designed membrane stretches across a particular cross-section at an established depth, this membrane being such that it moves with the acoustically induced fluid velocity within the hole in a manner as if were not present. The oscillating displacement of each such membrane is sensed optically, so that the output is a measure, in some sense, of the incident acoustic wave in the vicinity of the opening of the hole. Thus, one can view each hole as a separate hydrophone (the surrounding fluid is water) and the chip as a rectangular array of hydrophones.

The overall device considered here consists of an acoustic lens interfacing with the external environment and with the array lying in the focal plane of this lens (Fig. 2), the basic physics being analogous to an eyeball, with the MEMS array acting as a retina, each hydrophone acting as an optical nerve.

FIGURE 1. A MEMS chip with a rectangular array of square holes, each hole acting as a hydrophone element.

FIGURE 2. Plane wave obliquely incident on an acoustic lens in front of a MEMS array lying in its focal plane.

The effect of the lens is such that the acoustic disturbance focuses on a small area of the chip. If sound comes from a continuous range of directions, the mean squared output of each hydrophone corresponds ideally to the directional acoustic intensity (acoustic energy per unit time per sterradian) coming from the range of directions for which the lens tends to focus the sound on or near the aperture of that particular hydrophone. If the system works as intended, then the output of each hydrophone would be analogous to a single pixel in a digital image of whatever is scattering sound toward the general direction of this acoustic eyeball.

The Boston University MEMS-acoustics group has made a number of separate theoretical and experimental studies to determine the circumstances in which the device just described performs in a manner close to what
is ideally desired. A principal concern is crosstalk, whereby adjacent hydrophones are excited by the interaction of the acoustic field with another hydrophone.

**CROSS-TALK VIA DIFFRACTION**

One mechanism, and possibly the most important, is where the acoustically induced motion of fluid in one hole generates a secondary sound wave that reaches a second hole via diffraction and causes an acoustically induced fluid motion in that hole. To quantify the effect, consider a system with only two holes, distinguished by subscripts 1 and 2. Sound incident on hole number 1. in the absence of hole number 2, causes a fluid velocity with complex amplitude \( v_1(x_1,y_1) \) over the area \( A_1 \) of that hole. This motion in turn generates a diffracted wave back into the halfspace from which the incident wave is coming, the pressure amplitude of this diffracted wave being

\[
p_{\text{diff}}(x_1,y_1) = \frac{i\omega \rho}{2\pi} \int_{A_1} v_1(x_1,y_1) e^{ikR} dx_1 dy_1
\]

where the sense of the fluid velocity is into the hole \( +z \) and the integration is over the opening of hole 1. The distance \( R \) is from \( (x_1,y_1,0) \) to \( (x,y,z) \) with \( z \) being negative.

To determine the acoustically induced motion in hole 2, one regards the acoustic wave in equation 1 as being incident on hole 2. The mathematical model yields

\[
p_2(x_2,y_2) = 2p_{\text{diff},1}(x_2,y_2,0)
\]

\[
+ \frac{i\omega \rho}{2\pi} \int_{A_2} v_2(x'_2,y'_2) e^{ikR_2} R_2 dx'_2 dy'_2
\]

The dynamics of the acoustic field within the hole provide a third equation, this being a functional relation between \( p_2(x_2,y_2) \) and \( v_2(x_2,y_2) \). If one adopts the expedient artifice that an inflexible membrane of negligible mass floats at the opening of each hole, the membranes remaining parallel to the \( z = 0 \) plane, then this third relation takes the form

\[
F_2 = \int_{A_2} p_2 dx_2 dy_2 = Z_{\text{mech}}v_2 = iX_{\text{mech}}v_2
\]

where \( Z_{\text{mech}} \) is the mechanical impedance presented to external forces by the acoustical system within any given hole; \( X_{\text{mech}} \) is the corresponding mechanical reactance. For such circumstances, the area integral of Eq. 2 yields

\[
F_2 = \frac{ik\rho c}{2\pi} v_1 a^3 I(ka,k[x_2,c-x_1,c],k[y_2,c-y_1,c])
\]

\[
+ \frac{ik\rho c}{2\pi} v_2 a^3 J(ka)
\]

where \( a \) is the width of either square opening, and

\[
a^3 I = \int_{A_1} \int_{A_1} e^{i\omega R} dx_1 dy_1 dx_2 dy_2
\]

\[
a^3 J = \int_{A_2} \int_{A_2} e^{i\omega R} dx'_2 dy'_2 dx'_2 dy'_2
\]

The equations displayed above yield the result

\[
\frac{v_2}{v_1} = \frac{-2ka}{ka + 2\pi i[Z_{\text{mech}}/\rho c a^2]}
\]

The magnitude of this or the square of the magnitude serves as a measure of crosstalk for the envisioned system. It has a somewhat complicated frequency dependence and depends on the orientation (alignment) of the two holes as well as the distance between their centers. Determination of a numerical value requires in general the evaluation of the four-fold integrals involved in the quantities \( I \) and \( J \).

As an illustration of the deductions that the above formulation can yield, one can take the case where the separation between the two holes is large compared to the half-width of a hole opening and where, moreover, \( ka << 1 \), so that

\[
I \approx \frac{a}{r} e^{i kr}
\]

where \( r \) is the distance between the centers of the two holes. In this same limit, when \( ka << 1 \),

\[
J \approx 2.97 + i ka
\]

so that Eq. (7) reduces to

\[
\frac{v_2}{v_1} = \frac{-2ka}{ka[2.97] + 2\pi i[Z_{\text{mech}}/\rho c a^2] + i k^2 a^2 r^2 e^{i kr}}
\]

\[
\frac{ka[2.97] + 2\pi i[Z_{\text{mech}}/\rho c a^2] = 0}{[\text{Recall that } Z_{\text{mech}} \text{ is imaginary.]} \text{ At such a frequency, Eq. (7) yields}}
\]

\[
\frac{v_2}{v_1} = \frac{2i}{kr} e^{i kr}
\]

and the magnitude is consequently independent of the detailed design (including size) of the hole geometry.

Analogous simplicity results in the limit of very low frequency because the denominator in Eq. (7) is dominated by the impedance of the hole which, in this limit, is the same as that of a linear spring with spring constant \( \rho c^2 A^2 / V \), where \( V \) is the volume of the hole.

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Silicon Microphones and Earphones

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Recent progress in acoustic transducers made with the methods of silicon micromachining is reviewed. Of particular interest are silicon microphones based on the capacitive (condenser) principle and made on a single chip. Such transducers have been improved in several respects over the past few years and have recently found their first commercial applications. Also discussed are velocity-measuring silicon microphones based on the hot-wire principle as well as receivers and transmitters for the ultrasonic range. Finally, new work on piezoelectric silicon earphones for use in hearing aids is briefly reviewed.

INTRODUCTION

The first acoustic silicon transducers were described in 1983 [1,2]. These were a single-chip condenser micro-phone based on a sacrificial-layer technique [1] and a piezoelectric microphone made with a sputtered ZnO layer [2]. Over the following years, a variety of types of silicon microphones, utilizing condenser, piezoelectric, piezoresistive, modulated FET, optical-waveguide and other principles were discussed and realized [3-5].

After almost two decades of research and development, it appears that the condenser microphone is now the most advanced acoustic silicon transducer. However, research on other types of silicon microphones continues. These efforts are mostly directed toward the development of special-performance transducers applicable for specific tasks where other transducers can not be utilized.

In the present paper, progress in the field of acoustic silicon transducers, achieved over the past 3 years or so, is discussed. The emphasis is on single-chip condenser microphones. Due to constraints of space, only a small part of the recent work can be discussed. For more detail, the reader is referred to the literature.

CONDENSER MICROPHONES

Silicon condenser microphones have many advantageous properties and are therefore the type of acoustic silicon transducer most widely studied. Among their advantages are high sensitivity, large frequency range, low equivalent noise level, and low harmonic distortion. Most of the recently discussed condenser microphones are single-chip designs with low-stress membranes. Significant progress has been made toward the improvement of their design, their biasing and their packaging, as will be discussed in the following [6-17].

In the design of condenser microphones, the diaphragm is of particular importance since it determines to a large extent the sensitivity and resonance frequency of the device. In most cases, the diaphragm acts as a membrane and not as a plate; thus, its internal tension is of great importance. Since the tension of nitride membranes is usually too large, polysilicon membranes with low stress are now widely used [6-10]. With these, tensions of about 50 MPa, yielding resonance frequencies of about 100 kHz, can be readily achieved. Another means of reducing such tensions is a corrugated membrane design which has been occasionally employed [9,10]. Other low-stress membranes used are made of polysilicon-nitride double layers [11], specially deposited nitride [12], or p-doped silicon [13]. With proper care, sensitivities of about 10 mV/Pa and equivalent noise levels of approximately 24 dB(A) have been achieved. Another design feature is a differential transducer with the membrane sandwiched between two backplates [14]. Such microphones have the advantage of higher sensitivity and extended linear dynamic range.

Special attention has been devoted to the reduction of the applied bias voltage or to its replacement by an internal bias or an FM-modulation. The bias voltage necessary in such a system can be boosted from the low externally applied voltage (generally less than 2 V) to the required value (often above 10 V) by a dc-to-dc-converter [15]. The bias can be completely eliminated by the use of electrets. New developments consist in the use of nitride/oxide double-layer corrugated electret membranes with high and stable charge density [16] or in the application of a floating-electrode “electret” [9]. The latter consists of a floating electrode which is positioned between the two electrodes of the microphone and which can be charged by hot electrons generated by avalanche breakdown of a silicon p+n junction. This setup has the advantage that the floating electrode can be easily recharged. Lastly, an FM-modulation circuit, well known from conventional condenser microphones, has been implemented on a silicon substrate by standard CMOS processes before fabricating the silicon microphone on the same chip.
Such systems not only make dc-biasing unnecessary but also have the advantages of lower harmonic distortion, reduced dependence on variations of the power supply, and lower sensitivity to electromagnetic noise; this is, however, presently only possible at the expense of an equivalent noise level of 60 dB(A), which is high compared to the above-mentioned values.

An important aspect of silicon microphones is their packaging. This is complicated by the fact that often two or more chips have to be combined. Examples are the combination of a microphone and a backchamber chip or the additional inclusion of an ASIC chip connected to the microphone by an interconnect chip. Fully encapsulated systems with proper interfaces and with good electroacoustic properties, suitable for many applications, have recently been designed [11].

**OTHER TRANSDUCERS**

**Hot-wire microphones** are based on the anemometer principle and consist of two heated wires spaced at a small distance to each other. Due to the sound wave, gas particles will have different velocities at the two wires, and these will cool down differently. As has been realised long ago, this allows one to measure the particle velocity in the sound field.

Such microphones have been implemented in silicon some time ago [18]. They have recently been subjected to an optimization process [19]. The hot-wire microphone shows a high-frequency cutoff which was found to be due to two effects, namely heat conduction and the heat capacity of the wires [19,20]. For typical sensors, the cutoff frequency is of the order of 1 kHz.

**Ultrasonic silicon transducers**, both sound transmitters and sound receivers, have been of considerable recent interest. Some of the activity in this field has been devoted to capacitive transducers in the MHz range, used as immersion transducers or transducer arrays for medical diagnostics [21,22]. Due to their high resonance frequency, such transducers need only a very shallow air cavity and surface micromachining is very suitable for their fabrication. New work includes optimization with respect to electrode size [23] as well as efforts to characterize the arrays [22]. Another important step is the increase in transmitting sensitivity by decreasing the air gap to 50 nm by means of spacers on the lower side of the membranes [21]. Sound pressure levels of up to 5000 Pa have recently been achieved with this technique.

Piezoelectric and piezoresistive ultrasonic transducers have also been investigated recently. For example, a piezoresistive transducer has been used as a distance gauge in air by implementation of an acoustic Fabry-Perot interferometer [24]. With such systems, the distance range between 1 and 10 cm can be covered which is hard to achieve with other acoustic distance gauges.

Recently, **silicon earphones** based on the piezoelectric principle were studied [25]. These transducers consist of a polysilicon membrane onto which a thin (typically about 1 µm thick) PZT-layer is sputtered. Such layers are self-polarized and exhibit reasonable piezoelectric d33 constants. Sound pressures achieved in 2 ccm couplers are presently only 60 dB per Volt, but there is a large optimization potential by adjustments of resonance frequency and improvement of piezoelectric constants.

**CONCLUSIONS**

During the past few years, significant progress has been achieved in the design and performance of acoustic silicon transducers. Key areas where improvements have been realized are the equivalent noise level of silicon microphones, their packaging and their integration with the necessary integrated circuits. These aspects are important for their commercial introduction which is presently underway.

**REFERENCES**

Ultra acoustical waves scattering in the composite material reinforced by cylindrical crystal fibers

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Working out and applying of numerical-analytical method of solving problems of short elastic waves dispersion in parallel cylindrical reinforcing elements, fibers in the area of contact with elastic or viscous elastic isotropic matrix are introduced. Reinforcing fibers are represented as with circular and elliptic section filamentary crystals of orthorhombic class. The method is based on the representation of waves fields in the matrix and reinforcing fibers in rows due to basis particular waves equations solutions. Basis solutions of stationary waves equation for fibers are built in special non-classic vector functions of classic and generalized complex variables. The investigation of special physical and mechanical effects in reinforced medium was carried out. The effects of sharp localization of stresses in the fibers and the matrix close to contact surface under relative lengths of incident waves have been investigated.

NUMERICAL-ANALYTICAL METHOD OF INVESTIGATION OF WAVE FIELDS CHARACTERISTICS

The research of microstructure dynamic effects by propagation of elastic harmonic waves in fibrous composite materials with crystal fibrous is a new important scientific and applied problem and has many aspects. The possible way of its effective decision in two-dimensional and three-dimensional variant is based on a method of representation of wave fields in series on basic sets of the particular solutions of the dynamic equations for isotropic matrix binding and for a crystal material of fibrous. The factors of these series are defined from boundary conditions of contact of fibrous and binding. The description of such technique is given in the published reference. In the case of normal fall. The theoretical bases of a developed dispersion of a flat wave on a crystal fibrous of round section represented in [1,2].

Falling and scattered waves in a matrix binding

Relevant one-axes-armed by crystal fibrous of orthorhombic systems composite material are simulated as viscous-elastic medium in coordinate space $Ox_1,x_2,x_3$, which have set of long cylinders with collinear axes and equivalent round cross section $S_j$ with edges $\Gamma_j$. The inclusions are occupied regions $V_j = \{(x_1,x_2,x_3) \in S_j, -\infty < x_3 < x_3 < \infty\}$ in local coordinate systems, being in center of regions $S_j$ and having of collinear axis. The poles $O_j$ of local coordinate systems are disposed in subspace $x_3 = 0$ and geometrical centers of regions $S_j$. The fields of elastic displacement $\vec{U}^{(0)}$ and $\vec{U}^{(1)}$ in falling and scattered waves with frequency $\omega$ are characterized of its amplitude potentials $\Phi^{(m)}, \Psi^{(m)} (m = 0,1)$

$\vec{U}^{(m)} = \text{grad} \Phi + \text{rot} \Psi,$

$(\lambda + 2\mu)\Delta \Phi + \rho \omega^2 \Phi = 0,$

$\mu \Delta \Psi + \rho \omega^2 \Psi = 0, \quad \text{div} \Psi = 0,$

where $\lambda$ and $\mu$ - complex Lame modules for viscous-elastic medium. In the case of wave scattering on isolated fiber under arbitrary angle in plate $Ox_1,x_2$, amplitude wave potentials are presented in form

$\Phi^{(0)} = Ae^{i(\alpha n_1 x_1 + \beta n_2 x_2)},$

$\Psi^{(0)} = B_1 e^{i\beta(n_1 x_1 + n_2 x_2)}, \quad \Psi^{(1)} = \Psi^{(2)} = 0,$

$\Phi^{(1)} = \sum_\alpha A_n H^{(1)}_n (\alpha r) e^{i\varphi},$

$\Psi^{(1)} = \sum_\alpha B_n H^{(1)}_n (\beta r) e^{i\varphi}, \quad \Psi^{i} = \Psi^{(2)} = 0.$

Amplitude fields of elastic displacement and stresses for matrix haven representation

$U_1 = A_1 i n_1 \alpha e^{i(\alpha n_1 x_1 + \beta n_2 x_2)} + B_1 i n_2 \beta e^{i(\beta n_1 x_1 + \beta n_2 x_2)} + \sum_\alpha (\tau^{(\alpha)}_{11} H^{(1)}_{\alpha n} (\alpha r) + \tau^{(\beta)}_{11} H^{(1)}_{\beta n} (\beta r)) e^{i\varphi},$

$U_2 = A_1 i n_2 \alpha e^{i(\alpha n_1 x_1 + \beta n_2 x_2)} - B_1 i n_1 \beta e^{i(\beta n_1 x_1 + \beta n_2 x_2)} + \sum_\alpha (\tau^{(\alpha)}_{21} H^{(1)}_{\alpha n} (\alpha r) + \tau^{(\beta)}_{21} H^{(1)}_{\beta n} (\beta r)) e^{i\varphi},$

$\sigma_{11} = -A_1 \alpha^2 (\lambda + 2\mu) n_1^2 + \lambda n_2^2 \beta^2 e^{i(\alpha n_1 x_1 + \beta n_2 x_2)} - 2\mu B_1 \beta^2 n_1 n_2 e^{i(\alpha n_1 x_1 + \beta n_2 x_2)} + \sum_\alpha (\Delta^{(\alpha)}_{11} H^{(1)}_{\alpha n} (\alpha r) + \Delta^{(\beta)}_{11} H^{(1)}_{\beta n} (\beta r)) e^{i\varphi},$
\[ \sigma_{22} = -A_\alpha \alpha^2 \left( \lambda_{12} + \left( \lambda + 2 \mu \right) n_{12}^2 \right) e^{i\alpha(n_{12}^2 + n_{22}^2)} + 2 \mu B_\beta \beta^2 n_{12}^2 e^{i\beta n_{12}^2} \]  
\[ + \sum_{\nu} \left[ \Delta_{12}^\alpha H_n^{(1)}(\alpha r) + \Delta_{22}^\beta H_n^{(1)}(\beta r) \right] e^{i\nu \phi}, \]

\[ \sigma_{12} = -2 \mu A_\alpha \alpha^2 n_{12}^2 e^{i\alpha(n_{12}^2 + n_{22}^2)} + \mu B_\beta \beta^2 (n_{12}^2 - n_{22}^2) e^{i\beta n_{12}^2} \]  
\[ + \sum_{\nu} \left[ \Delta_{12}^\alpha H_n^{(1)}(\alpha r) + \Delta_{12}^\beta H_n^{(1)}(\beta r) \right] e^{i\nu \phi}. \]

Here
\[ \tau_{(r)} = -\frac{1}{2} \alpha (i)^{-1} \left( A_{s+1} + (-1)^i A_{s-1} \right), \]
\[ \tau_{(\theta)} = -\frac{1}{2} \beta (i)^{-1} \left( B_{s+1} + (-1)^i B_{s-1} \right), \]
\[ \Delta_{(\nu)}_{12} = -\left( \lambda + 2 \mu \right) \alpha^2 A_{s+1} + \frac{1}{2} \left( \lambda + 2 \mu \right) \alpha^2 \left( A_{s+2} - A_{s-2} \right), \]
\[ \Delta_{(\nu)}_{22} = \frac{1}{2} i\mu \alpha \left( A_{s+2} - A_{s-2} \right), \]
\[ \Delta_{(\nu)}_{12} = -\left( -1 \right)^i \frac{1}{2} i\mu \beta \left( B_{s+2} - B_{s-2} \right), \]
\[ \Delta_{(\nu)}_{22} = -\left( -1 \right)^i \frac{1}{2} \mu \beta \left( B_{s+2} - B_{s-2} \right). \]

Wave field in armed fibrous

The wave field of elastic displacement in fibrous is described by series
\[ U^{(2)}_j = \sum_{\nu} \sum_{m=1}^{2} D_{pm} \tilde{\Psi}_{pmj}, \]
\[ \tilde{\Psi}_{pmj} = 2 \text{Re} \left[ \tilde{\Psi}_{pm} (z, z_m) + \tilde{\Psi}_{pm} (z, \bar{z}_m) \right] \]

on the basis particular solutions of two-dimensional equations
\[ \tilde{L} \tilde{\Psi} = \tilde{M} \tilde{\Psi}, \]

of dynamic deformation for elastic medium of orthorhombic class. The basic solutions
\[ \tilde{\Psi}_{pm} (z, z_m) = \left\{ \Psi_{pm1} (z, z_m), \Psi_{pm2} (z, z_m) \right\}, \]

in this case are submitted by non-classical complex vector functions
\[ \tilde{\Psi}_{pm} (z, z_m) = \sum_{k=0}^{\infty} \sum_{s=0}^{p} \tilde{G}_{sk}^{(k,p,m)} z^{2k+s} z_m^{-s} \frac{\Gamma(2k + s + 1) \Gamma(p - s + 1)}{\Gamma(2k + s + 1) \Gamma(p - s + 1)}. \]

Here \( \tilde{G}_{sk}^{(k,p,m)} \) - vector factors described by obvious recursion formulas
\[ \tilde{G}_{sk}^{(k,p,m)} = \tilde{M}_{1m} - \tilde{M}_{2m} - \tilde{M}_{3m} - \tilde{M}_{4m}, \]
\[ (s = 0, p; \quad k = 1, \infty), \]
\[ \tilde{M}_{1m} = \tilde{M}_{2m} = \tilde{M}_{3m} - i\tilde{M}_{4m}, \]
\[ \tilde{M}_{2m} = 2 \tilde{M}_{2m} + (i + \mu) \tilde{M}_{4m} - 2i \mu \tilde{M}_{4m}, \]
\[ \tilde{M}_{3m} = \tilde{M}_{2m} + \mu \tilde{M}_{2m} + \sigma \tilde{M}_{4m}, \]
\[ \tilde{M} = \Omega^2 \tilde{E}, \quad \Omega^2 = \rho \omega^2 R^2 C^{-1}, \]
in which
\[ z = x_1 + i x_2, \quad z_m = x_1 + \mu x_2, \]
\[ \mu_m - \text{roots characteristic equation} \]
\[ \det \| \tilde{M}_{1m} + \mu \tilde{M}_{2m} + \mu^2 \tilde{M}_{4m} \| = 0, \]
\[ \tilde{E} \text{ - identity matrix, } \tilde{M}_{1}, \tilde{M}_{2}, \tilde{M}_{4} \text{ - matrisses of factors in representation of the matrix operator} \]
\[ \tilde{L} = \tilde{M}_{1} \tilde{E}^2 + \tilde{M}_{2} \partial_1 \partial_2 + \tilde{M}_{4} \partial_2^2 \]

of the equations of movement.

For fields \( U^{(2)}_j \) on a contour of a fibrous is carried out the orthogonal decomposition on angular coordinate. It allows from edge conditions on border of contact of a fibrous and matrix to obtained of systems of the linear algebraic equations for factors \( A_{s}, B_{s}, D_{pm} \) and to receive the formulas, through which is carried out of detailed physical-mechanical analysis of the investigated phenomenon.

**CALCULATIONS**

On Figures 1, 2 respectively the distributions of voltage arising on border of contact of a fibrous and a matrix at length of a wave, falling along an axis \( OX_2 \), equal to a diameter of a fibrous by \( (\lambda + 2 \mu) / 2 \mu = 1.5, \quad C_{22} / C_{11} = 10 \) and \( C_{22} / C_{11} = 0.1 \) are presented.

**FIGURE 1.** \quad **FIGURE 2.**

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Thermoelastic Loss in Silicon Resonators

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The internal friction as a function of a high Q mode in a single-crystal silicon double paddle oscillator is examined from the perspective of known fundamental mechanisms. The loss at high temperature (above 60 K) is found to be in agreement with theoretical prediction of loss due to thermoelastic dissipation due to transverse thermal currents. The theory employs a combination of finite element and analytical models. The importance of this dissipation mechanism as a function of scale is briefly discussed. We find that the relative importance to this mechanism scales with the size of the structure and that for NEMS size structures it is less important than other intrinsic loss mechanisms.

Difficulties have arisen in the process of creating high-Q microelectromechanical systems (MEMS); e.g. RF components where the Q's are found to be lower than expected from scaling considerations of fundamental loss mechanisms\textsuperscript{1,2}. The paddle is fabricated out of 300 \textmu m c-Si and is attached to an Invar block.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Diagram}
\caption{A sketch of the Double Paddle Oscillator (DPO). The paddle is fabricated out of 300 \textmu m c-Si and is attached to an Invar block.}
\end{figure}

In order to improve our understanding of the loss mechanisms at work in silicon based oscillators and, more generally, to achieve higher Q versions of MEMS oscillators, we have investigated the dynamics and loss mechanisms of a macroscale double paddle oscillator (DPO) (Fig. 1) made of single crystal Silicon.

The temperature dependence in the measured Q\textsuperscript{-1} for our DPO has been reported previously\textsuperscript{3}. Shown in Figure 2 is a plot of the Q\textsuperscript{-1} as a function of temperature for a number of different experiments associated with the DPO. At the lowest temperature, 4 K, the loss is approximately 2 \times 10\textsuperscript{-8}. At 60 K, the loss sharply increases and approaches 1.5 \times 10\textsuperscript{-5} at room temperature.

In order to understand the above behavior, we investigated the known intrinsic absorption mechanisms for crystalline silicon. Phonon-phonon interactions lead to an inverse of the quality factor given by Bömmel & Dransfeld\textsuperscript{4}, as,

\begin{equation}
Q^{-1} = \frac{CT\gamma^2}{\rho V^2} \frac{\omega \tau_{ph}}{1 + (\omega \tau_{ph})}
\end{equation}

where $\gamma$ is the Greneisen's constant, $C$ is the heat capacity per unit volume, $T$ is the temperature, $\rho$ is the density, $V$ is the longitudinal sound velocity, $\omega$ the angular frequency. This last quantity is the phonon relaxation time given by,

\begin{equation}
\tau_{ph} = \frac{3\kappa}{C V_D^2}
\end{equation}

where $\kappa$ is the thermal conductivity and $V_D$ the mean Debye sound velocity. At the frequencies of interest for the DPO, $\omega \tau_{ph} \ll 1$. Comparing these values for Q\textsuperscript{-1} to the measured values for the paddle oscillator in Fig. 1 (frequency 5.3 kHz), we find the observed Q\textsuperscript{-1} levels to be two or more orders of magnitude greater than the levels due to phonon-phonon scattering limits.

\begin{equation}
Q^{-1} = \begin{cases} 
3.3 \times 10^{-10}, & T = 300^\circ K \\
7.4 \times 10^{-11}, & T = 4^\circ K
\end{cases}
\end{equation}

The second phenomena we examined is the purely classical effect of thermoelastic dissipation in which inhomogeneous dynamic strains induce temperature gradients and subsequent heat flow. Landau and Lifschitz\textsuperscript{5} give Q for longitudinal strains as

\begin{equation}
Q^{-1} = \frac{\kappa T \alpha^2 \rho \omega}{9C^2}
\end{equation}

where $\alpha$ is the thermal expansion coefficient. Inserting parameters relevant to Silicon\textsuperscript{5} we find

\begin{equation}
Q^{-1} = \begin{cases} 
9.5 \times 10^{-13}, & T = 300^\circ K \\
3.3 \times 10^{-11}, & T = 4^\circ K
\end{cases}
\end{equation}

Again, for frequencies on the order of 5,300 Hz, these Q\textsuperscript{-1} levels are much lower (over four orders of magnitude) than that of our DPO.
Finally, we considered thermoelastic dissipation associated with transverse strain as treated by Zener for the case of a vibrating thin beam or reed. This relaxation due to the flow of transverse thermal currents leads to an inverse quality factor given by

\[ Q^{-1} = \frac{E\alpha^2 T}{C} \frac{\omega_0}{1 + (\omega_0\tau_{th})^2} \]

where \( E \) is Young's modulus and the thermal relaxation time is given in terms of the oscillator thickness, \( a \), by

\[ \tau_{th} = \frac{a^2 C}{\pi^2 K} \]

The maximum \( Q^{-1} \) associated with this mechanism is

\[ Q^{-1} = \begin{cases} 5.8 \times 10^{-5}, & T = 300^\circ K \\ 10^{-15}, & T = 4^\circ K \end{cases} \]

At, or near, room temperature the maximum dissipation is quite high in contrast to internal friction associated with either phonon-phonon interactions or thermoelastic damping of compressional motion.

The quantity in Eq. 6 is a maximum when the resonant angular frequency of the device is equal to the inverse of the relaxation time (~1859 Hz for the DPO). Further, the quantity in Eq. 6 is also a maximum value for the case when the motion in question is pure bending. We determine the relative amount of transverse versus total strain energy and multiply this “participation factor” by the maximum \( Q^{-1} \) given by Eq. 6. This ratio was found by use of a finite element model of the DPO and the application of theory detailed in Williams. For the high Q second antisymmetric mode of the paddle oscillator we found this to be approximately 0.13. We then compute the \( Q^{-1} \) curve (solid curve) versus temperature shown in Fig. 2. The overall agreement at the higher temperatures in the levels as well as the shape suggests strongly that thermoelastic dissipation due to transverse strains plays a role in determining Q above 60 K.

We developed broadband predictions of the maximum amount of dissipation possible for c-Si resonators that are constructed of 1.5 \( \mu \)m, 50 \( \mu \)m, and 50 nm thick material. These results are shown in Fig. 3 together with a similar prediction for the macroscale DPO (300 \( \mu \)m) and the two other two intrinsic damping mechanisms. Examination of Fig. 3 suggests that this mechanism may play a dominant role in the overall loss in the system for MEMS scale devices (Si thicknesses of \( \approx 1 \mu \)m and resonant frequencies ~100 MHz). However, examination of Fig. 3 also shows that this mechanism indeed becomes less important at NEMS scales and frequencies. Moreover, internal damping due to phonon-phonon interactions should increase in importance at NEMS scales. This work support by ONR.

REFERENCES

6. Properties of Silicon - EMIS Datareview
Application of Surface Acoustic Wave Device to High Performance Linear Motor


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We have proposed to utilize a surface acoustic wave (SAW) devices for ultrasonic motors and demonstrated its possibility. A multi contact type slider using a silicon slider whose contact surface is micro machined have been proposed. Diameter and density of projections fabricated on silicon sliders were examined experimentally. Importance of elastic deformation control to obtain large output force with a surface acoustic wave motor is discussed in this paper. By adding pre-load to slider, stator and slider surfaces are deformed a few tens nano meter. Appropriate deformation in normal direction against normal vibration displacement amplitude of surface acoustic wave existed. By moderate deformation, the output force of the surface acoustic wave motor was enlarged up to about 10 N and no-load speed was 0.7 m/sec. By energy circulation drive, the efficencis was improved. Nano meter order step motion of this motor is also investigated. Possibility of sub nano step motion is discussed.

POINT OF DISCUSSION

A schematic view of the surface acoustic wave motor[1] is illustrated in Fig. 1. RF electrical power (9.6 MHz) is transduced to elastic wave. The Rayleigh wave is excited at the interdigital transducer (IDT) with piezoelectric effect. By traveling wave propagation, the surface particles of the SAW device move in elliptical motion. Since the amplitude of the elliptical motion is 10 nm order, the contact condition of the slider is very critical. To control the contact condition, namely, the elastic deformation of the slider and the stator surface in nano meter order, a lot of projections are fabricated on a slider surface as shown in Fig. 2. The projection diameter was 20 mm.

In static condition, the elastic deformation and stress were evaluated with Hertz contact model in case of a single steel ball slider [2]. From this calculation and the simulation result [3], the friction drive mechanism of surface acoustic wave is illustrated as shown in Fig. 3. The wave crest is distorted, hence the elasticity has influence on the friction drive condition[4, 5]. Elastic deformation of the stator surface beneath the projection from the initial position were evaluated with the FEM at static condition. The maximum depression changed in proportion to the pre-load and contact pressure. In 4x4 mm² square area, the sliders had projections from 1089 to 23409. Depression value of the stator surface were 30 nm to 50 nm against 100 N pre-load.

DRIVING PERFORMANCE

Mechanical output of the motor such as no-load speed and output force were measured at the driving voltage of 125 V. At this condition, the vibration amplitude in normal to the stator surface was 21 nm. The vibration velocity of horizontal direction was 1.1 m/sec. The pre-load was changed up to 110 N. Hence, the maximum depression was changed up to about 2 times of the vibration amplitude.
No-load speed were plotted against the ratio of the maximum depression to the vibration displacement amplitude as shown in Fig. 9. No-load speed decreases with increase of the maximum depression, namely, increase of the pre-load. At low pre-load condition below the ratio of 0.5, the speed saturated. The maximum speed was 88% of the vibration velocity using 23409 projections slider.

The output force depended on the depression in spite of the projection density were different as shown in Fig. 10. The maximum output force was obtained when the maximum depression was from 1.0 to 1.5. This phenomenon was followed by different vibration amplitude. The maximum output force was 10 N when the highest projection density slider was tested.

Different diameter size sliders were tested. The diameters were from 5 mm to 50 mm. These sliders showed similar characteristics as 20 mm diameter sliders. Namely, the maximum output force was obtained when the maximum depression was almost same as vibration amplitude.

We also measured the stepping motion of the motor. When we drove 40 burst waves, the stepping displacement was about 8 nm as shown in Fig. 6. The driving time was only 4 µs. Much fine steps was available, but due to measurement technical matter, the sum nano motion steps has not confirmed yet.

**CONCLUSION**

The relation between the static elastic deformation by the pre-load and the mechanical output of the motor was examined. We found that the elastic deformation of the whole slider area have influence on the friction drive of the surface acoustic wave motor. We will improve the friction drive model to represent the experimental results discussed here. The stepping motion of 8 nm was measured.

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**REFERENCES**

A Miniature Actuator Using Simple-Beam-Mount Piezoelectric Bimorph for High-Resolution Imager Module

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A new high resolution CCD imager has been developed. A CCD chip is swung synchronously at frame frequency in horizontal direction with the aid of newly designed piezoelectric bimorph actuators. In order to improve the deflection characteristics of a Simple-Beam-Mount piezoelectric bimorph actuator, piezoelectric bimorph mounting methods were investigated. A new Simple-Beam-Mount piezoelectric bimorph actuator with U-shaped metal plates at each end was designed. It was found that the displacement for a new Simple-Beam-Mount piezoelectric bimorph actuator was approximately 4 times greater than that for a conventional piezoelectric bimorph actuator. By using this piezoelectric bimorph actuator for the swinging operation, which is twice higher than that with a conventional imaging operation, has been realized without increasing sensor pixels.

INTRODUCTION

In order to ensure high quality reproduced pictures, resolution improvement is required for the solid-state image sensor. By moving the sensor relative to the incident image it is possible to increase the image sampling density without increasing pixels in the sensor.\cite{1,2} Double enhancement on horizontal resolution has been realized by periodically vibrating a thin glass plate in front of a sensor.\cite{3}

The authors have fabricated a high-resolution CCD module on which a CCD is swung synchronously at signal frame frequency with the aid of piezoelectric bimorph actuators. In this high resolution CCD module, in order to improve the bimorph actuator displacement sensitivity, piezoelectric bimorph actuator mounting methods were investigated.

This paper presents improvement in mount structure for a piezoelectric bimorph used in a highly sensitive deflection actuator. A piezoelectric bimorph, mounted on a free moving axis, such as by a U-shaped metal plate at both ends of the piezoelectric bimorph, was found to be very effective in improving the element's sensitivity. Also, details on constructing high-resolution CCD modules using the new mounting method for piezoelectric bimorph are described in this paper.

OPERATION PRINCIPLE

Figure 1 shows the swinging operation principle. The unit pixels are composed of a photo sensing section and a signal read-out section optically shielded. As can be seen in this figure, the CCD chip is periodically swung at a signal frame frequency (rectangular wave (30 Hz)) right and left in a horizontal direction. Signal charges stored in the photo sensing area are transferred to the read-out area during the vertical blanking period by applying read-out pulses. The CCD module having swinging operation produces a signal frame picture composed of a signal stored in one field time at one site and that stored in another field time at the new site. This is equivalent to an increase in the number of picture elements in the horizontal direction. Also, image picture signals obtained in fields A and B are relatively shifted in a horizontal direction on a monitor so as to be spatially agreed with the sampling points on the sensor, resulting in a horizontal resolution enhancement by a factor of two.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Swinging operation Principle.}
\end{figure}
PIEZOELECTRIC BIMORPH

The rectangular bimorph element consists of two piezoelectric thin plates with electrodes to both surfaces, electro statically prepolarized in the thickness direction and bonded together in series in the polarization direction, with a very thin conductive medium. When an electric field was applied across each thickness direction, the one plate length increases in the length direction because of the reverse polarization and the electric field direction, while the other plate length decreases. Consequently, the bimorph bends. The bimorph alternately vibrates when an AC voltage is supplied to electrodes.

In order to improve the displacement sensitivity, a simple-beam-mount piezoelectric bimorph with U-shaped metal plates at each end was designed.

Figure 2 shows a cross section view of two kinds of mounting structures. Categories A and B are newly designed structure and conventional structure, respectively. Piezoelectric ceramic plate dimensions are 25mm(length) x5mm (width) x0.1mm (thickness). The thin metal plate used in the central medium is a SUS sheet, 0.1mm thick.

Figure 3 shows displacement at the center of the bimorphs as a function of the applied AC voltage. As seen in this figure, the displacement for Category A, mounted by the U-shaped metal plates, is approximately 4 times larger than that for conventional bimorph, Category B.

A piezoelectric bimorph element mounted on a free-moving axis, such as a U-shaped metal plate, was found to be very effective in improving the element's sensitivity and to be most suitable for swing operation.

DEVICE STRUCTURE

The CCD module construction is shown in Figure 4. A ceramic substrate having a CCD chip and a flexible plate are mounted on two piezoelectric bimorph actuators, which are arranged in parallel with each other. Both ends of these bimorph elements are fixed in the box shaped package. By applying pulse voltages to these bimorph element electrodes, the two-bimorph element plates are vibrated with the same swinging response in the same direction, resulting in the CCD chip swinging in a horizontal direction on an image format. The box shaped CCD module size is about 30mm x 20mm x 11mm, in which the device height is 11mm. The ceramic dual-in-line package has a flexible plate.

CONCLUSIONS

A high resolution CCD module, driven with two piezoelectric bimorph actuators, has been fabricated, resulting in double enhancement on horizontal resolution.

Also, a highly sensitive bimorph deflector, using a new mounting method with U-shape metal plates on both ends, has been fabricated. The displacement was approximately 4 times larger than that for conventional bimorph 's mounting. Therefore in order to vibrate a PH/2 swinging range, the voltage applied to the bimorph elements is the same as the IC driving voltage.

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Sound Radiation Modeling of Rectangular Plate with Piezoelectric Actuators and Sensors for Active Structural Acoustic Control

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Modeling of sound radiation and vibration of thin rectangular plate is described in this paper for providing with reliable and versatile designing tool for Active Structural Acoustic Control (ASAC) system development. This paper proposes application of Finite Element Method (FEM) model for describing the bending vibration of elastic plate and the sound radiation. This modeling is applied as radiation filter to designing the control system. The piezoelectric elements may be bonded to the plate as sensors and actuators in the control system, where generation of bending moment by an actuator and of electric charge by a sensor is evaluated. ANSYS code is applied to a typical rectangular plate, size of which is 270[mm] by 310[mm] with 1.0[mm](thickness), containing each of three pieces of piezoelectric element for sensors and actuators. The amplitude distribution of bending vibration in the plate measured by LDV agrees well with the computation by ANSYS. The result is applied to evaluate the sound radiation from the plate.

INTRODUCTION

Recently examples of installing Active Noise Control system in a commercial airplane are reported. Technology for eliminating periodic sound has been established. Main goal to realizing the Active Noise Control system in the plane shifts into eliminating broadband noise excited by the sound wave radiated from the jet engine, or excited by pressure fluctuation in the turbulent boundary layer on the surface of the fuselage. Active noise control technology which can treat the broad band noise above mentioned shall be called Active Structural Acoustic Control (ASAC) because of including the distributed sensors and actuators in the structure control system. At designing and optimizing the active control system for the vehicle noise problem, the ASAC approach is essential for the nature of the control object. Demand to model the control object correctly leads to introduce the Finite Element Model (FEM), because of its versatile capability of modeling the plant. A rectangular plate with pieces of piezoelectric element of sensors and actuators is supposed as a model of control plant in this paper. The sound radiation model is deduced on the basis of the dynamic model by the FEM model.

MODELING CONTROL SYSTEM

Figure 1 shows schematic illustration of the experimental setup as a model of control plant described in this paper. Thin plate with pieces of piezoelectric element as sensor and actuator on both surfaces is fixed on the top of the duct in the bottom of which a loud speaker is installed. Bending vibration in the plate excited by the incident sound wave from the loud speaker causes the radiation.
modeled with Radiation filter, the inputs to which are the mode velocities, and the output from which is distribution of the sound radiation pressure in the outer free space. The radiated sound power is evaluated by summing up the sound pressure.

**RADIATION FILTER**

The shapes of the measured amplitude distribution as shown in Figures 3(a) and (b) suggest resemblance between the shape resulted from the simple support boundary condition and the shape calculated with FEM with the clamped boundary condition. In case of the simple support, the shape is expressed by the sum of sinusoidal function, therefore the sound radiation field is described by the simple analytic functions[2]. As shown in Figure 3(b), the shape by FEM displays the local roughness resulted from inhomogeneity of the bending stiffness in the plate. The shape function from FEM is nevertheless expressed by the series of sinusoidal functions. This expansion of the shape function leads to the simple analytic description of the sound radiation field. Radiation filter with rather simple function is deduced by the idea of the expansion of the shape function.

**CONCLUDING REMARKS**

The designing method for ASAC system by FEM was proposed. In order to prove the effectiveness of the method a F.B. control system is designed.

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Variational Model for a MEMS Waveguide of Varying Cross-Section

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A variational formulation for the modeling of the acoustic field inside the cavity of a MEMS hydrophone is presented. Special care is dedicated to the implementation of the boundary conditions at the opening of the device, where the reradiated field is described using a Rayleigh integral. The resulting mixed spectral-element/finite-element method solves the mode-coupling problem in a numerically stable way.

PROBLEM STATEMENT

A miniaturized Silicon hydrophone, obtained via anisotropic etching of Si wafers, is studied. The cavity of the MEMS hydrophone delimits a fluid-filled waveguide of variable square cross-section which is embedded in an ideally infinite Silicon baffle, as depicted in Figure 1. The Silicon can be treated as rigid within a reasonable degree of approximation. One end of the waveguide, located at \( z = 0 \), is open to the incident sound field. The width of the opening is \( 2a \). The other end of the waveguide, located at \( z = L \), is closed by a planar Silicon surface, and the incident acoustic field consists of a plane wave of angular frequency \( \omega \) which propagates from negative infinity to the origin along the positive direction of the \( z \) axis.

For the problem parameters associated with the MEMS application the acoustic wavelength is of the same size as the waveguide dimensions. It is thus necessary to develop a technique which does not rely on asymptotic approximations of the wavefield. This is achieved with the help of a variational approach.

SOLUTION METHOD

With \( e^{-i\omega t} \) time dependence assumed throughout this paper the acoustic field is described by the Helmholtz equation

\[ \nabla^2 p + k^2 p = 0. \tag{1} \]

The acoustic pressure is \( p \), and \( k = \frac{\omega}{c_a} \), where \( c_a \) is the soundspeed. The \( z \) component of the acoustic Euler equation

\[ i\rho c_a k v_z = \frac{\partial p}{\partial z} \tag{2} \]

is also taken into account. Here \( v_z \) is the component of the acoustic fluid velocity directed along the \( z \) axis and \( \rho \) is the fluid density.

Applying the variational calculus to equations (1) and (2) yields a variational statement of the form

\[ \delta \left\{ \int_V L(p, \nabla p, v_z) dV + \int_{A_0} M(p, v_z) dA \right\} = 0. \tag{3} \]

The functional \( L \) is defined over the interior volume \( V \) of the waveguide, while \( M \) is defined at the opening \( A_0 \) which is located at \( z = 0 \). Because of the rigid boundary conditions the integrals over the walls and the closed end of the waveguide vanish. The boundary conditions at the opening are written using a Rayleigh radiation integral:

\[ p(x, y, 0) = 2P_0 + \frac{i\omega\rho_0}{2\pi} \int_{A_0} v_z(x_0, y_0, 0) e^{ikR_0/R_0} dA_0, \tag{4} \]

where \( R_0 = \sqrt{(x-x_0)^2 + (y-y_0)^2} \), \( dA_0 = dx_0 dy_0 \), and \( P_0 \) is the pressure amplitude of the incident plane wave.

FIGURE 1. The MEMS hydrophone as waveguide of varying cross-section inside an infinite rigid baffle. Although it is not clearly visible in this figure, the angle between the planes of the lateral walls and the plane of the opening, \( z = 0 \), is taken to be \( \arccos(\sqrt{1/3}) \approx 54.74^o \), which is the angle that is generated by the anisotropic KOH etching of the Silicon.
The above integral extends over the area of the opening. The boundary condition (4) accounts for the incident wave as well as for the field reradiated out of the MEMS cavity, and it defines an impedance operator \( M^{-1} \) which is a representation of the inverse of the operator \( M \). Using the spectral-element formulation presented below it is possible to build the boundary condition explicitly into the variational statement via a matrix equation.

A spectral-element formulation is introduced by describing the acoustic field using the representations

\[
\begin{align*}
    p &= P_0 \sum_n f_n(z) \Phi_n(x,y|z), \quad (5) \\
    v_z &= \frac{i \rho_0 c}{\rho} \sum_n q_n(z) \Phi_n(x,y|z), \quad (6)
\end{align*}
\]

where \( f_n(z) \) is an axially dependent dimensionless modal pressure amplitude, \( q_n(z) \) is a dimensionless modal \( z \)-velocity amplitude, and \( 0 < z < L \). Each \( \Phi_n(x,y|z) \) is a linearly independent solution of the \( z \)-separated eigenproblem \( (\partial^2/\partial x^2 + \partial^2/\partial y^2 + \kappa_n(z)^2) \Phi_n(x,y|z) = 0 \), with \( (\partial \Phi_n/\partial n_{T,x})/\partial x + (\partial \Phi_n/\partial n_{T,y})/\partial y = 0 \) on the perimeter of the waveguide cross-section. The vector \( (n_{T,x},n_{T,y},0) \) is normal to the perimeter, and it lies in the \( xy \)-plane. The eigenfunctions \( \Phi_n \) form a complete set, and a solution that satisfies the boundary conditions at the rigid walls, which are generally not perpendicular to the \( xy \) plane, can be determined by inserting equations (5) and (6) into the variational formulation (3). In a practical implementation one retains only a finite number of eigenfunctions, and consequently the variational principle satisfies the boundary conditions only approximately. Because of the geometrical symmetry of the problem, only the first \( N \) symmetric eigenfunctions are considered in the discrete implementation of the method.

In order to stabilize the numerical algorithm, the modal coefficients \( f_n(z) \) and \( q_n(z) \) are interpolated with piecewise linear functions between a discrete set of nodes \( 0 = z_1 < z_2 \ldots < z_j < \ldots z_5 = L \). Substituting the interpolated coefficients into equation (3) and varying the unknown pressure-amplitudes, \( f_{n,j} = f_n(z_j) \), for each mode at each node together with variation of the velocity-amplitudes, \( q_{n,j} = q_n(z_j) \), yields the following system of linear algebraic equations:

\[
\begin{bmatrix}
    f_{1,1} \\
    \vdots \\
    q_{N,5}
\end{bmatrix} = \mathbf{b} \neq 0. \quad (7)
\]

The matrix \( [A] \) is sparse and band-diagonal, and the vector \( \mathbf{b} \) stems from the boundary condition at \( z = 0 \). For the studied MEMS application, the typical matrix dimensions are \( 2SN \times 2SN \approx 2000 \times 2000 \), and the solution time is below 10 minutes on a standard desktop workstation.

**RESULTS AND CONCLUSIONS**

In Figure 2 the coupling of energy which occurs between the different modes can be clearly seen. The waveguide is filled with water. The narrowest cross-section is \( 124\mu m \) wide, and it is located at \( z = 200\mu m \), in correspondence of node 80. The frequency of the field is \( f = 5.5MHz \). The results were obtained using the first 10 symmetric basis functions. As an effect of the mode-coupling, the fluid velocity components increase in correspondence of the narrowest cross-section. The rigid-wall boundary condition at the closed end (node 100) is clearly satisfied by the solution, since all \( |q_n(L = 250\mu m)| \) vanish.

The presented variational technique describes the acoustic field inside cavities of varying cross-section for which the waveguide dimensions are comparable to the wavelength. Using the outlined method the behavior of a MEMS hydrophone can be predicted, and the design of a device can be optimized by performing numerical experiments with a wide range of parameters.

**ACKNOWLEDGMENTS**

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![FIGURE 2. Plot of modal velocity amplitude coefficients \( |q_n(z)| \) versus axial position, obtained using 10 modes and 100 nodes. Mode \( n = 1 \) is the plane wave mode, represented by the continuous line. The dashed line corresponds to mode 2 and the dotted line represents mode 3.](image-url)
Super-resolution Optical Vibrometry Applied to MEMS Oscillator

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As in optical microscopy, the spatial resolution for an optical vibrometry system is limited by diffraction, and for visible light is between 500 nm and 1 µm. An alternative microscopy technique known as Near-field Scanning Optical Microscopy or NSOM circumvents this problem by illuminating a target sample with a sub-wavelength optical source held in close proximity to the sample surface so that the sample is in the optical near-field of the sub-wavelength source. A super-resolution image is formed by raster-scanning the source over the sample. This technique has now been extended to vibrometry by forming an interferometer with the light collected from the sample surface. The interferometer is used to infer a local velocity of the sample surface. Small single paddle MEMS oscillators are used to demonstrate the high resolution capability of the instrument.

\textbf{INTRODUCTION}

Single-crystal silicon mechanical oscillators are finding applications in both fundamental research and applications. In the process of developing these oscillators into microelectromechanical systems (MEMS), certain limitations and restrictions hindered the rapid development of new applications. The most serious challenge comes from the low Q of MEMS oscillators\cite{1,2}, which is far below the theoretical limit\cite{3}. In order to understand the performance of the systems, we must understand their dynamics.  

For larger MEMS structures we have used laser Doppler Vibrometry (LDV) to identify modes\cite{4}. In this paper we develop a new technique using Near-field Scanning Optical Microscopy or NSOM to study the dynamics of structures too small for the limitations of conventional LDV. The idea of super-resolution optical imaging dates to the late 1920’s\cite{5}, but was not realized until the mid 1980’s\cite{6}. We extended the NSOM technique to vibrometry by forming an interferometer with the light collected from the sample surface. To demonstrate the high resolution capability of the instrument, we used the new device to determine the frequency response curve for small single paddle MEMS oscillators.

\textbf{NSOM APPARATUS}

The NSOM achieves super-resolution by illuminating a surface with a sub-optical-wavelength source in close proximity to the surface. The sub-wavelength source is realized by launching laser light into an optical fiber with the exit end pulled to a very sharp taper of about 100 nm. The fiber tip is metalized so that the light can only emerge from the 100 nm aperture. The close proximity of the fiber tip to the sample, necessitated by diffraction, is maintained by an analog controller\cite{7} in the same way as an atomic force microscope does. A photodetector collects light that emanates from both the sample surface and the tip itself. The two light sources are separated by a distance that is less than the resolution of the collecting system. This ensures that the two will interfere.

The fiber tip is bounded to the side of a quartz tuning fork that is excited electrically at its resonance (~33.6 kHz). The sample surface is moved toward the tip by three piezo-electric stack actuators. The tuning fork response is damped when the vibrating tip is within approximately 50 nm of the sample (fig. 1). This is sensed by monitoring the current from the tuning fork. This current serves as the input to the feedback controller that maintains the separation gap by continuously adjusting the voltage that drives the piezo actuators.

\textbf{FIGURE 1.} NSOM optical fiber tip is held within 50nm of sample surface. A collecting lens system mixes light from the tip and sample on the surface of a photodetector.
MEMS OSCILLATOR

The oscillator presented here is a free plate suspended by two arms (fig. 2). Analytic predictions of the natural frequencies of five modes, where the arms provide restoring forces and the plate is an inertial mass, are given in table 1 [7].

The portion of the silicon wafer that holds the oscillator is mounted on an ultrasound transducer by an aluminum post. The ultrasound transducer is driven by a 200 ns pulse. This drive excites many modes.

The frequency response curve shows two peaks at the predicted natural frequencies of the oscillator (fig. 3). This spectrum is the FFT of the ensemble average of 5000 acquisitions. Resonance peaks of the out-of-plane translation and simple torsion modes are seen at 6.8 and 7.7 MHz respectively. The extraneous peak at 600 kHz is believed to be the dominant response of the backing structure of the ultrasound transducer. When the NSOM tip is moved off the oscillator, the 600 kHz peak remains and the 6.8 and 7.7 MHz peaks disappear. The in-plane modes are not seen because this device is sensitive to only one component of motion.

**FIGURE 2.** (a) Electron microscope image of oscillator. (b) Dimensions of oscillator are a = 4.08 μm, b = 4.08 μm, c = 3.91 μm, d = 250 nm, and w = 200 nm.

**FIGURE 3.** Frequency spectrum of the optical signal.

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